THE DISTRIBUTION AND EFFECTS OF FALLOUT IN LARGE NUCLEAR-WEAPON CAMPAIGNS†

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The purpose of this paper is to provide a simple way of evaluating the consequences of radioactive fallout from a large nuclear-weapon campaign without resorting to detailed map studies Simple analytic formulas based on numerical data in Rand Memorandum RM-1969 are presented that enable one to make rapid statistical estimates of the distribution of fallout and its effects on a population, as well as the consequences of changing the total delivered yield or the targeting doctrine The method was checked against a detailed map study, using the same parameters, and was found to be in excellent agreement with its results A method of optimally distributing weapons among large areas in order to maximize radiation casualties is deduced on the basis of the formulas, and curves are exhibited expressing the casualties produced as a function of total yield delivered The achievement of optimized attacks does not require a delivery accuracy with probable error less than about a hundred miles In addition, the formulas are applied to a number of other targeting doctrines, and the resulting curves of casualties versus total delivered yield are presented

THIS PAPER takes as its point of departure a model analyzed by The Rand Corporation ‡ This model is as follows Nuclear weapons are dropped uniformly at random into a large area Each weapon is equally likely to fall at any point in the area, independently of where any of the others may fall It is further assumed that the weapons burst simultaneously at the surface, that all are of the same yield, and that the area is large compared to the area of contamination of a single weapon

We shall show that the results of Rand's analysis of this model are adequately summarized by a single simple analytic expression, giving the probability distribution of the integrated dose 24 hours after detonation (the H_r+24 integrated dose) at any point in the area as a function of the yield density (megatons/square mile) delivered in the area

We then observe that the response function of a population (the fraction of casualties produced as a function of the unshielded dose in roentgens)

† The paper presented here is a mathematical study which was conducted independently by the authors It does not represent and is not intended to represent official thinking or plans

‡ S M Greenfield, "Radioactive Contamination from a Multibomb Campaign," Rand RM-1969, January, 1956

can usually be expressed as a simple analytic form over a wide range of conditions This function is then combined with the fallout distribution function to produce a single formula expressing the expected casualty fraction in a region as a function of the yield density delivered to the region

Having determined the expected response as a function of delivered yield, we turn our attention to the question of 'optimizing' the distribution of a fixed stockpile in order to maximize radiation casualties Equations are deduced that relate the yield density to be delivered in each region to the population density in the region, for such an 'optimal' attack

Finally the formulas are applied to a number of targeting doctrines, namely

1. Yield density distributed optimally
2. Yield density proportional to population density
3. Yield density uniform over entire area under attack
4. Air-base targeting

For each of these doctrines, curves are displayed relating expected casualties under various assumptions of population preparedness to the total yield committed

LIST OF SYMBOLS

H_r = time of weapon burst

$x = H_r + 24$-hr integrated dose, roentgens

X = randon variable associated with x

y = natural logarithm of $H_r + 24$-hr integrated dose $y = \ln x$

Y = random variable associated with y, $Y = \ln X$

μ = mean of Y

σ^2 = variance of Y

ζ = mean of X

τ^2 = variance of X

θ = yield density scale factor

D = yield density in neighborhood of a point (megatons/10^4 sq naut mi)

D_0 = that yield density for which $\sigma^2 = \ln 2 = \sigma_0^2$

μ_0 = mean of Y for yield density D_0

$R(x)$ = fraction casualties produced by 24-hr integrated dose x

$\bar{R}(D)$ = fraction casualties produced in region of yield density D

ξ, η = parameters of $R(x)$ depending upon casualty definition and population preparedness

Φ = standard cumulative normal function $\Phi(y) = \left(1/\sqrt{2\pi}\right)\int_{-\infty}^{y} e^{-x^2/2}\, dx$

χ = dimensionless parameter defined by $\chi = (\mu - \xi)/\sqrt{\sigma^2 + \eta^2}$

ρ = population density (people/sq naut mi)

$\Gamma(\rho)$ = population density distribution

A = total area under attack

$D^0(\rho)$ = yield density as a function of population density which maximizes casualties for fixed total yield expended

E = total expected casualties
$\bar{E}$ = average expected casualties per unit area
S = total yield expended in campaign
$\bar{S}$ = average yield density
λ = Lagrange multiplier
ρ_0 = population density cut-off, for optimal attacks regions of population density below ρ_0 are not attacked at all
D^* = the yield density of highest efficiency, i e , for which $\bar{R}(D)/D$ is a *maximum*, for optimal strikes, no region is attacked with density lower than D^*
$\bar{R}^*$ = fraction casualties produced by D^* $\bar{R}^* = R(D^*)$

SUMMARY OF RESULTS

BEFORE presenting the detailed arguments we shall give a summary of the final results of this study These results are as follows

Fallout Distribution

For a large campaign, in which there is significant overlapping of fallout patterns, the probability distribution of the 24-hr integrated dose x (roentgens) at a particular point, is a *lognormal distribution* That is, the natural logarithm of the dose, $y = \ln x$, is normally distributed with mean μ and variance σ^2

$$P(y)\,dy = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{-(y-\mu)^2}{2\,\sigma^2}\right\} dy,$$

where μ and σ^2 depend only on the yield density D (in megatons/10^4 sq naut mi for weapons that are ⅔ fission) delivered in the region of the point

$$\sigma^2 = \ln(1 + D_0/D),$$
$$\mu = \mu_0 + \tfrac{1}{2}\ln 2 - \tfrac{1}{2}\sigma^2 + \ln(D/D_0),$$

where μ_0 and D_0 are empirical constants

Response Functions

A response function R expresses the fraction casualties produced in a population as a function of the dose received by the population It depends both upon the definition of casualty, and upon the condition of the population (such as amount of shelter available, warning time, etc)

Under a wide range of conditions the response functions are closely approximated by *cumulative normal functions* of the *logarithm*, y, of the H_r+24-hr integrated dose x That is, if a population has received the total H_r+24-hr dose x, the fraction casualties produced, R, is

$$R = \Phi[(y-\xi)/\eta],$$

where $y = \ln x$ and $\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-x^2/2}\,dx,$

and ξ and η are constants depending upon the casualty definition and population condition

Combined Formula

When the fallout distribution function is combined with the response function, the result is a function $\bar{R}(D)$ expressing the expected casualty fraction $\bar{R}$ in a region as a function of the yield density D delivered in the region

$$\bar{R}=\Phi[(\mu-\xi)/\sqrt{\sigma^2+\eta^2}]$$

Optimized Attacks

An 'optimal' attack strategy is a function $D^0(\rho)$, the yield density to be delivered to each region as a function of the population density of the region, which maximizes the total casualties for a fixed total yield expenditure An optimal strategy has the property that there is a certain *cut-off population* density, ρ_0, such that regions of lower population density are not attacked at all, while regions of higher density are always attacked with yield density not less than a certain minimum D^* In particular, for a strategy $D(\rho)$ to be optimal it must satisfy

$$D(\rho)=0 \qquad (0\leqq\rho<\rho_0)$$

$$d\bar{R}/dD=(\rho_0/\rho)(\bar{R}^*/D^*) \qquad (\rho_0\leqq\rho\leqq\infty)$$

where

$$\bar{R}^*/D^*=\max_D[\bar{R}(D)/D]$$

These equations define a class of optimal strategies parameterized by the population density cut-off ρ_0 By solving them for a selection of values of ρ_0, and computing the total casualties and yield expended in each case, one arrives at the casualty versus total yield function for optimal attacks

MATHEMATICAL DEDUCTIONS

We now discuss in detail the derivation and justification of the formulas summarized above

Fallout Distribution Function

Let us begin by making the assumption (to be tested later) that, for any yield density D, the H_r+24-hr integrated dose at a point is a random variable X which is *lognormally* distributed

$$P(x)\,dx=(1/x\sigma\sqrt{2\pi})\exp[-(\ln x-\mu)^2/2\sigma^2]\,dx \qquad (1)$$

Our present task is to determine the dependence of the parameters μ and σ^2 on D Let us define ζ,τ^2 to be the mean and variance, respectively, of X

$$\zeta=\int_0^\infty x\,P(x)\,dx, \qquad \tau^2=\int_0^\infty (x-\zeta)^2\,P(x)\,dx \qquad (2)$$

Then straightforward calculation yields the relations

$$\zeta = e^{(\sigma^2/2+\mu)}, \qquad \tau^2 = e^{2\mu}\,[e^{2\sigma^2} - e^{\sigma^2}] = \zeta^2\,[e^{\sigma^2} - 1], \tag{3}$$

for which the inverse relations are

$$\sigma^2 = \ln(\tau^2/\zeta^2 + 1), \qquad \mu = \ln(\zeta^2/\sqrt{\zeta^2+\tau^2}) = \ln\zeta - \tfrac{1}{2}\,\sigma^2 \tag{4}$$

Now suppose that for a single attack, of a particular yield density, the mean and variance of X are ζ and τ^2 respectively Then if we make, instead of a single attack, θ attacks, the new mean and variance will be given by

$$\zeta' = \theta\zeta, \qquad \tau'^2 = \theta\tau^2, \tag{5}$$

according to the laws for sums of independent random variables (The random variable X' for the multiple attack is simply the sum of the corresponding variables X for each single attack) But θ independent strikes are completely equivalent to a single strike of density θ times greater than the density of the original attack Therefore, equations (5) express the general density scaling law, when θ represents the ratio of the densities and is an integer Furthermore, since this law must hold for all ways of decomposing a strike of a given density into substrikes, we can conclude that it holds for all real numbers θ as well as for integers

Equations (5), which express the laws of scaling the mean and variance of X for different yield densities, together with the relations (3) and (4), imply the following scaling laws for μ and σ

$$\sigma'^2 = \ln\left(\frac{e^{\sigma^2} - 1 + \theta}{\theta}\right), \qquad \mu' = \mu + \tfrac{1}{2}\,(\sigma^2 - \sigma'^2) + \ln\theta, \tag{6}$$

where $\theta = D'/D$

We must now comment upon a mathematical difficulty of this scheme While we have correctly deduced the scaling laws for μ and σ under the *assumption* that we always have a lognormal distribution, using the laws for the addition of random variables, it is unfortunately *not* true that the sum of two lognormally distributed random variables is lognormally distributed (The convolution of two lognormal distributions is not lognormal) However, over the range of interest in our case the discrepancies are sufficiently small that they may be safely ignored, and it is a good approximation to regard all of our distributions as lognormal

Let us now, for convenience, define a constant D_0 to be that yield density (as yet unknown) such that $\exp\sigma_0^2 = 2$ for the corresponding σ, and let μ_0 denote the corresponding value of μ Then (6) can be rewritten in the form

$$\sigma^2 = \ln(1 + D_0/D), \qquad \mu = \mu_0 + \tfrac{1}{2}\ln 2 - \tfrac{1}{2}\,\sigma^2 + \ln(D/D_0) \tag{7}$$

These equations contain only two constants, μ_0 and D_0, which must be evaluated empirically, and we have succeeded in our endeavor to determine the dependence of μ and σ upon D

In order to determine the values of μ_0 and D_0, a number of points were read from Rand RM-1969 (Fig 1) for each of seven selected values of D(5, 10, 15, 20, 30, 40, and 50 meg/10^4 sq naut mi), and the log dose versus fractional coverage plotted on probability paper Straight lines were visually adjusted through these points, for each D, and the resulting means and variances read off By using the first of equations (7), a value of D_0

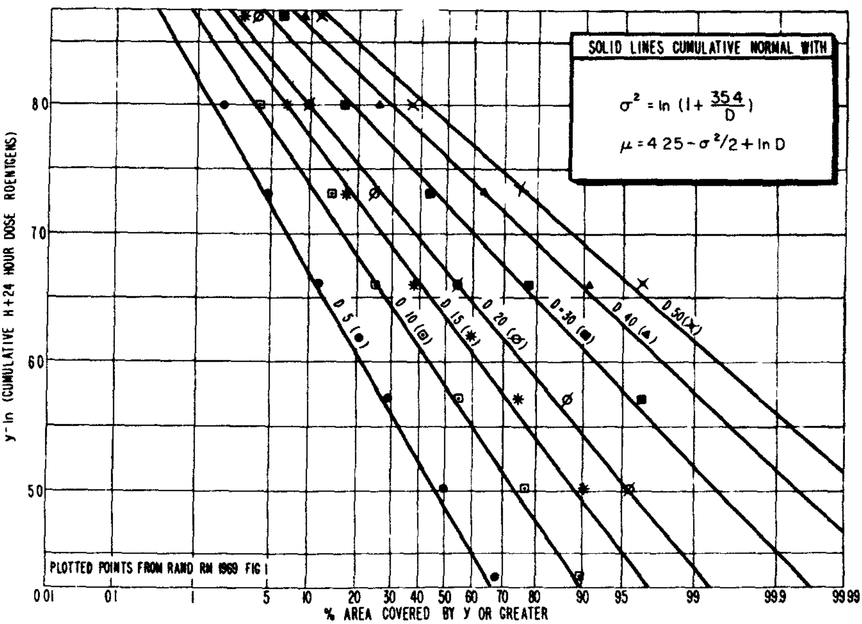

Fig 1 Fraction of area covered by doses for which the natural logarithm is greater than y Curves are plotted for several values of the yield density, D, which is indicated in megatons per (100 naut mi)^2

was computed for each value of D, and an appropriate average selected for best fit in the region $D = 10$ to 20 The resulting choice was

$$D_0 = 35\ 4 \text{ megatons}/10^4 \text{ sq naut mi} \qquad (8)$$

Having decided upon D_0, μ_0 was evaluated by the second of equations (7), and again an appropriate average selected

$$\mu_0 = 7\ 47 \qquad (9)$$

The resulting equations (from 7) are then

$$\sigma^2 = \ln(1 + 35\ 4/D), \qquad \mu = 4\ 5 - \tfrac{1}{2}\sigma^2 + \ln D \qquad (10)$$

Figure 1 shows the theoretical fractional coverage curves based upon equations (10), with points from Rand RM-1969 plotted to show the adequacy of the fit Figure 2 graphically depicts equations (10), from which values of μ and σ^2 may be read off for any value of yield density D

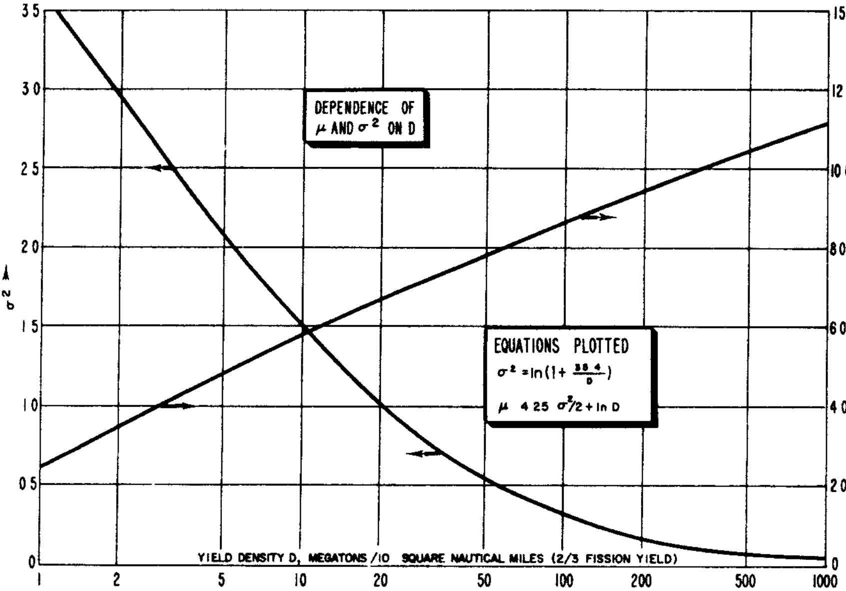

Fig 2 Curves showing the computed dependence of μ and σ^2 on yield density, D

Response Functions

Let us now assume that the *response function*, R, for a population has the form of a cumulative normal function of the logarithm of the $H+24$-hr integrated dose, y

$$R(y)=\Phi[(y-\xi)/\eta], \tag{11}$$

where R is the casualty fraction, ξ and η are constants depending upon the condition of the population and definition of casualty, and Φ is the standard cumulative normal function defined by

$$\Phi(z)=\frac{1}{\sqrt{2\pi}}\int_{\infty}^{z}\exp(-\tfrac{1}{2}\,\omega^2)\,d\omega \tag{12}$$

If equation (11) is plotted on *probability paper* it is a straight line Thus, the assumption that response functions are represented by (11) can be tested by simply plotting log(H_r+24-hr dose) against *casualty fractions produced* for *known* response functions, and observing whether or not the points are fitted by a straight line

To illustrate the fit of arbitrary response functions to the lognormal form we have chosen two sets of time-average-shielding factors that are based on behavior patterns chosen by some social scientists for two particular situations that were under consideration The first behavior pattern (called the 'unprepared case') is intended to represent the behavior and consequent average shielding which might be obtained by an unprepared population given only a few hours warning of attack The 'prepared case' is intended to represent the time-average-shielding which might be obtained by a population given six-months alert to prepare and build shelters

The original assumptions were based on consideration of both the available shielding and the fraction of time the population might be expected to take advantage of the shielding However, we will list here only the time-average-shielding

The time-average-residual numbers† for both cases are given below

Unprepared		*Prepared*	
Percent population	Residual number	Percent population	Residual number
52	0 50	13	0 50
6	0 40	8	0 20
7	0 29	5	0 18
31	0 24	26	0 15
2	0 07	7	0 07
2	0 017	30	0 038
		11	0 016

These numbers are based on the behavior of large groups that were given identical behavior The numbers are quite arbitrary, and certainly do not have any simple analytic form However, the response functions calculated from these behavior patterns are so smoothed by statistical variations in biological response that the results fit quite acceptably to the lognormal curve Both casualties and deaths were computed for sixty days after initial exposure

Figure 3 shows the result of plotting these data on probability paper, with the natural logarithm of the dose as ordinate The straight lines which are drawn through the plotted points give a remarkably good fit, justifying the assumption that equation (11) represents response functions

† Residual numbers are a decimal representation of the fraction of the unshielded dose that reaches the personnel in question

The parameters ξ and η^2 were determined from Fig 3, with the following results

Case	*Response*	ξ	η^2
Unprepared	Total casualties	6 00	0 34
	Deaths	6 62	0 34
Prepared	Total casualties	7 32	1.32
	Deaths	8 01	1 32

(13)

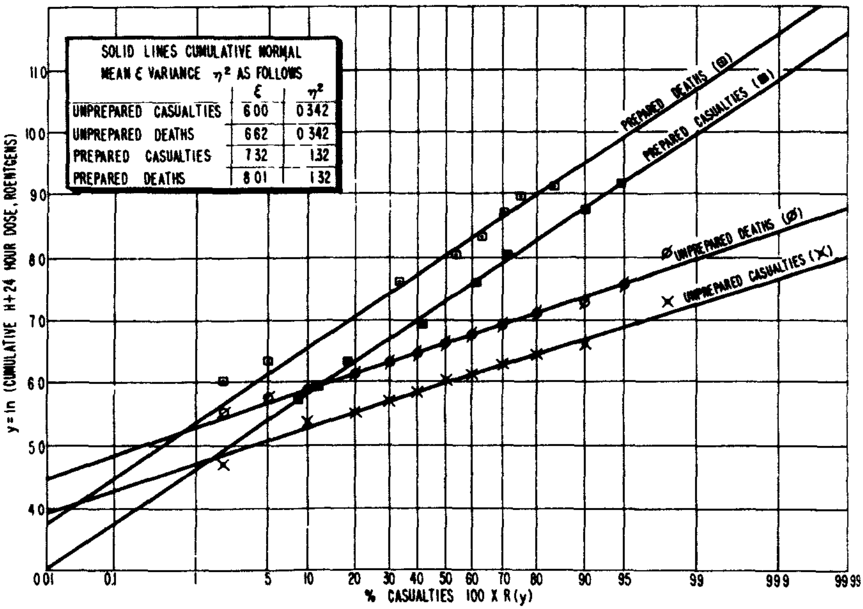

Fig 3 Total population response at 60 days plotted against the natural logarithm of the unshielded dose

Combined Formula

From equation (1) we can deduce the probability distribution of y

$$P(y)dy = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(y-\mu)^2}{2\sigma^2}\right] dy, \qquad (14)$$

so that the log-dose is normally distributed, with mean μ and variance σ^2 given by (10)

Since we also have the response function $R(y)$, given by (11), we can compute the over-all expected casualty fraction $\bar{R}$ (casualty probability)

$$\bar{R} = \int_{-\infty}^{\infty} R(y)\, P(y)\, dy$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(y-\mu)^2}{2\sigma^2}\right] \Phi\left(\frac{y-\xi}{\eta}\right) dy = \bar{R}(\mu,\sigma,\xi,\eta) \tag{15}$$

A little manipulation greatly simplifies this equation Writing (15) in full we have

$$\bar{R} = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(y-\mu)^2}{2\sigma^2}\right] \int_{-\infty}^{(y-\xi)/\eta} \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-\nu^2}{2}\right] d\nu\, dy \tag{16}$$

By making the transformations $\omega = (y-\mu)/\sigma$ and $\alpha = \nu - (\sigma\omega + \mu - \xi)/\eta$, interchanging the order of integration, and carrying out the integration over ω we obtain

$$\bar{R} = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi(1+\sigma^2/\eta^2)}} \exp\left[\frac{-(\eta\alpha+\mu-\xi)^2}{2(\eta^2+\sigma^2)}\right] d\alpha, \tag{17}$$

and a final transformation $\Psi = (\eta\alpha + \mu - \xi)/\sqrt{\eta^2+\sigma^2}$ gives

$$\bar{R} = \int_{-\infty}^{(\mu-\xi)/\sqrt{\sigma^2+\eta^2}} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}\Psi^2)\, d\Psi = \Phi\left(\frac{\mu-\xi}{\sqrt{\sigma^2+\eta^2}}\right) \tag{18}$$

Thus, our final formula for the expected casualty fraction takes on the simple form

$$\bar{R}(\mu,\sigma,\xi,\eta) = \Phi[(\mu-\xi)/\sqrt{\sigma^2+\eta^2}] = \Phi(\chi), \tag{19}$$

which is a function of the single dimensionless parameter χ

$$\chi = (\mu-\xi)/\sqrt{\sigma^2+\eta^2} \tag{20}$$

Equation (19), together with relations (10), then summarizes the expected casualty fraction as a function of the yield density of the attack D, and the constants ξ, η, of the population condition, given by (13)

The expected response $\bar{R}$ is shown as a function χ in Fig 4, which is a graph of equation (19)

Finally, the expected response is plotted directly as a function of yield density in Fig 5, which represents the basic results to be used in the remainder of the paper

Optimal Distribution of Weapons

Now that we have determined the casualty fraction as a function of yield density for prescribed population conditions and casualty definitions, a natural question arises For a given population, distributed geographi-

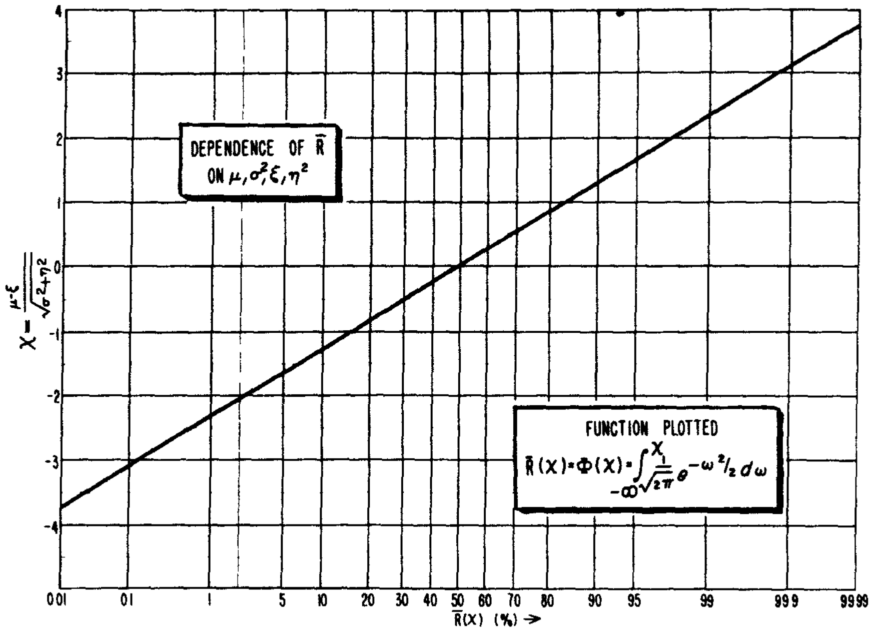

Fig 4 Dependence of total casualty fraction $\bar{R}(\chi)$ on the dimensionless parameter χ

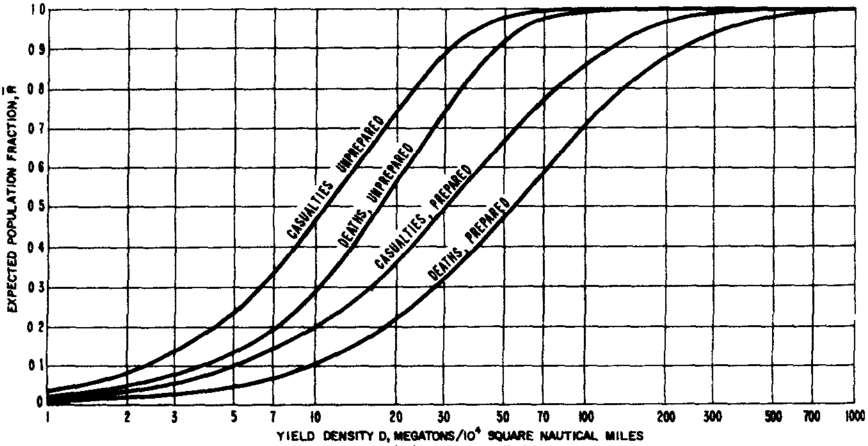

Fig 5 Expected population response at 60 days as a function of yield density, D, of attack

cally in some known manner, how should one distribute a fixed number of weapons in order to maximize the expected casualties?

The population distribution can be represented by a population density function, ρ, (a function of geographical position), for which the value at a particular position is taken to be the average population density of some suitably chosen neighborhood at the point

Our problem is, therefore, to determine a function $D(\rho)$, the yield density to be delivered in each region as a function of the population density, which maximizes the expected casualties subject to the constraint that the total megatonnage delivered is a fixed quantity S

Let us now define a function $\Gamma(\rho)$, to be called the *population density distribution,* such that the fraction of area contained in the region whose population density is between ρ and $\rho+d\rho$ is $\Gamma(\rho)\,d\rho$ Letting A stand for the total area under attack, we have that for any choice of $D(\rho)$ the total expected casualties produced is

$$E=\int_0^\infty \bar{R}[D(\rho)]\,\rho A\,\Gamma(\rho)\,d\rho, \tag{21}$$

or, normalizing out the total area

$$\tilde{E}=E/A=\int_0^\infty \bar{R}[D(\rho)]\,\rho\,\Gamma(\rho)\,d\rho \tag{22}$$

On the other hand, the total yield expended in the campaign is easily seen to be

$$S=\int_0^\infty D(\rho)\,A\,\Gamma(\rho)\,d\rho, \tag{23}$$

or, again normalizing

$$\tilde{S}=\frac{S}{A}=\int_0^\infty D(\rho)\,\Gamma(\rho)\,d\rho \tag{24}$$

Our task is therefore to choose $D(\rho)$ in order to maximize $\tilde{E}$ subject to the constraint (24) Introducing a Lagrange multiplier λ, in the usual manner, reduces the problem to maximizing the single (unconstrained) expression

$$\begin{aligned}\tilde{E}&=\int_0^\infty \bar{R}(D)\,\rho\,\Gamma(\rho)\,d\rho+\lambda\left[\tilde{S}-\int_0^\infty D(\rho)\,\Gamma(\rho)\,d\rho\right]\\&=\int_0^\infty [\bar{R}(D)\,\rho-\lambda D]\,\Gamma(\rho)\,d\rho+\lambda\tilde{S}\end{aligned} \tag{25}$$

Before proceeding, we list several considerations that will be pertinent in the subsequent derivation

1 The following properties of our functions are easily verified D is nonnegative, $\bar{R}(D)$ is monotone increasing, $\bar{R}(0)=0$, $\bar{R}(\infty)=1$, $d\bar{R}/dD$ is nonnegative, bounded above, and $[d\bar{R}/dD]_{D=0}=0$

2. It is obvious, but can be rigorously proven that if $D(\rho)$ is optimal, then it must be monotone increasing with ρ (It never pays to put the higher yield densities in areas of lower population density)

3. Since $D(\rho)$ is, in effect, a freely chosen strategy it may very well be discontinuous

We proceed by calculating the variations produced by a variation of δD in D

$$\delta\tilde{E}=\int_0^\infty \left[\rho\frac{d\bar{R}}{dD}-\lambda\right]\delta D\ \Gamma(\rho)\ d\rho \tag{26}$$

Now we must notice a fact peculiar to our case, namely, that the nonnegativity of D implies that the variation δD is not *arbitrary* in case D itself is zero, but is constrained to positive values (while if $D>0$, δD can have arbitrary sign, of course) Now the necessary condition for maxima is simply that $\delta\tilde{E}\leqq 0$ (not the more restrictive $\delta\tilde{E}=0$), and we see from (26) that, to satisfy this criterion, either

$$D=0 \qquad \text{or} \qquad \rho\, d\bar{R}/dD=\lambda, \qquad (0\leqq\rho\leqq\infty) \tag{27}$$

since for $D=0$ the integrand is *negative* (λ nonnegative, $d\bar{R}/dD=0$) and δD is restricted to positive values, which produce negative variations in $\tilde{E}$, while for $D>0$, δD is arbitrary and the integrand must vanish

But these conditions (27) cannot be met by a continuous function $D(\rho)$, because $d\bar{R}/dD$ is nonnegative and bounded above and $\lim_{D\to 0} d\bar{R}/dD=0$, so that (27) cannot have any solution involving arbitrarily small values of D, and $D(\rho)$ must possess a *jump discontinuity* from zero at some point ρ_0

Therefore, there exists a ρ_0 such that $D(\rho)$ is discontinuous at ρ_0, and satisfies

$$\begin{aligned} D&=0, && (0\leqq\rho<\rho_0) \\ \rho d\bar{R}/dD&=\lambda && (\rho_0\leqq\rho\leqq\infty) \end{aligned} \tag{28}$$

But now we have introduced a new parameter ρ_0, which must itself be chosen so as to maximize $\tilde{E}$, which can now be written

$$\tilde{E}=\int_{\rho_0}^\infty [\bar{R}(D)\ \rho-\lambda D]\ \Gamma(\rho)\ d\rho+\lambda\tilde{S} \tag{29}$$

From which we compute

$$d\tilde{E}/d\rho_0=-\Gamma(\rho_0)\ \{\bar{R}[D(\rho_0)]\ \rho_0-\lambda\ D(\rho_0)\} \tag{30}$$

And applying the condition that $d\tilde{E}/d\rho_0=0$, we obtain the final condition for our optimal program

$$\bar{R}[D(\rho_0)]/D(\rho_0)=\lambda/\rho_0 \tag{31}$$

We can use (31) to evaluate λ in (28), so that the conditions for an optimum distribution $D(\rho)$ become

$$\begin{aligned} d\bar{R}/dD &= (\rho_0/\rho)\{\bar{R}[D(\rho_0)]/D(\rho_0)\}, && (\rho_0 \leqq \rho \leqq \infty) \\ D &= 0 && (0 \leqq \rho < \rho_0) \end{aligned} \qquad (32)$$

And we notice that at ρ_0, (32) implies that

$$d\bar{R}/dD = \bar{R}/D \quad \text{at} \quad \rho = \rho_0, \qquad (33)$$

which is the condition that $\bar{R}/D$ is a *maximum*, which we shall denote by

$$\bar{R}^*/D^* = \max_D [\bar{R}(D)/D] \qquad (34)$$

Summarizing (32), (33), (34), we can state the rules for the optimum distribution $D(\rho)$ as follows

For every choice of a population density cut-off ρ_0 there is an optimal distribution $D(\rho)$ which has the property that $D(\rho)$ is zero for all densities less than ρ_0, while areas of the critical density ρ_0 are attacked with yield density D^* (producing the maximum value of $\bar{R}/D$—the highest 'efficiency'), and for population densities $\rho > \rho_0$, $D(\rho)$ is chosen to satisfy $d\bar{R}/dD = (\rho_0/\rho)\, \bar{R}^*/D^*$ Symbolically,

$$\begin{aligned} D &= 0, && (0 \leqq \rho < \rho_0) \\ d\bar{R}/dD &= (\rho_0/\rho)\bar{R}^*/D^*, && (\rho_0 \leqq \rho \leqq \infty) \end{aligned} \qquad (35)$$

where $\bar{R}^*/D^*$ is given by (34)

Equations (35) give optimal distributions for each value of population density cut-off ρ_0 For each choice of ρ_0 the optimal distribution may be computed, and from it, the casualties E and the total invested yield S By performing this calculation for a number of choices of ρ_0, the general dependence of E on S for optimal distribution of weapons is obtained

In order to illustrate the dependence of yield density on population density for an optimized campaign, we shall carry out the optimization of *deaths* in the *unprepared* case, and, for comparison, in the prepared case as well

From equation (35) we see that

$$\rho/\rho_0 = (\bar{R}^*/D^*)/(d\bar{R}/dD) \qquad (36)$$

The function $d\bar{R}/dD$, as a function of D, can be determined by differentiation of equations (19) and (10) with the appropriate parameters from (13) for the deaths in each case The maximum value of $\bar{R}/D$, $\bar{R}^*/D^*$, is then determined as the point where $d\bar{R}/dD = \bar{R}/D$, and occurs at about $D^* = 14$ megatons/10^4 sq naut mi for the unprepared case and at $D^* = 23$ megatons/10^4 sq naut mi for the prepared case

Having determined the right-hand side of (36) as a function of D one is able to make a plot of ρ/ρ_0 vs D for optimal distributions This plot is presented for the present cases in Fig 6

To use Fig 6 one first selects a population density cut-off, ρ_0 The optimal distribution of weapons is then to place *no* weapons in regions of population density less than ρ_0, and in regions of higher population density, $\rho > \rho_0$, to place them with density given by Fig 6 It should be noted that

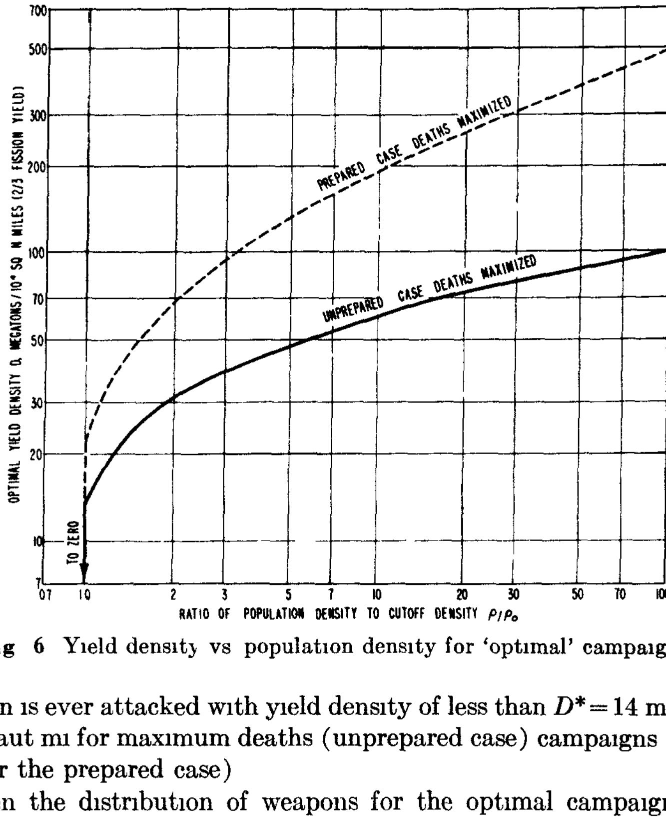

Fig 6 Yield density vs population density for 'optimal' campaigns

no region is ever attacked with yield density of less than $D^* = 14$ megatons/10^4 sq naut mi for maximum deaths (unprepared case) campaigns (twenty-three for the prepared case)

Given the distribution of weapons for the optimal campaign arising from a particular choice of ρ_0, it is necessary to calculate both the total casualties (by applying the Fig 5 to each region) and the total yield expended By performing this calculation for a number of selections of ρ_0, one can make a direct plot of casualties produced versus total yield expended for optimal campaigns

ILLUSTRATIVE EXAMPLES

We shall now apply the formulas developed in the previous sections to a number of concrete cases to illustrate the usefulness of the methods

We shall exhibit response curves (casualty fractions) as a function of total delivered yield for the entire population of the U S A under various assumptions about targeting doctrine and the degree of preparation of the population

Two sets of shielding factors were used The first, or unprepared case was chosen by social scientists to be representative of the shelter which might be used by an untrained populace given emergency instructions to remain under shelter after attack The prepared case represents a behavior which might exist in a well-trained population given six months to build shelters on an emergency basis The two cases are chosen to indicate the sensitivity of the results to changes in shielding and exposure patterns which can result from civilian defense measures

The different targeting philosophies that we shall consider here are

1. Density of drop optimized (in accordance with the methods discussed) to produce maximum radiation deaths
2. Density of drop proportional to population density (A general attack on production, transportation, and communication facilities would probably coincide closely with this case)
3 Density of drop uniform over entire country
4. Air-base targeting

In order to calculate casualties for a particular type of strike we divide the total area into sub-areas receiving different yield densities The formulas are then used to obtain the percentage response for each area to its corresponding yield density

In our particular case, the United States was treated as forty-eight separate states, except that New York was broken ⅔, ⅓ by area into an upper and lower New York State

Strictly speaking, the compiled results correspond to a random drop of weapons in each sub-area They therefore indicate the results that can be obtained by a delivery system which allows one to hit a chosen state without more accurate specification of the impact point This is roughly comparable to a probable delivery error of about 100 miles

DISCUSSION OF VARIOUS TARGETING DOCTRINES

Optimized Drop

Figures 7 and 8 show the results of an optimized drop The calculation is based on the distribution of population in the forty-eight states The resulting casualty curves are those that can be expected with a delivery mechanism having a probable delivery error of about one hundred miles

It is not possible to optimize more than one quantity at a time In Fig 7 we optimized deaths in the unprepared case The remaining curves are not optimized but show the expected casualties that would occur in the other cases when the attack is optimized for unprepared deaths For comparison, Fig 8 shows the result of optimizing for the prepared deaths

It is worth remarking that the optimized drop strategy has the peculiar

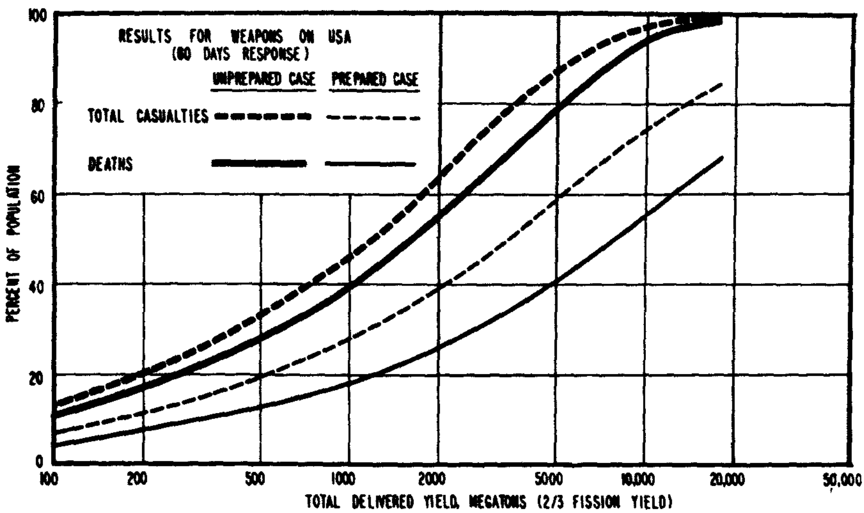

Fig 7 Population response in the United States resulting from 'optimal' attacks designed to *maximize deaths in an unprepared population*

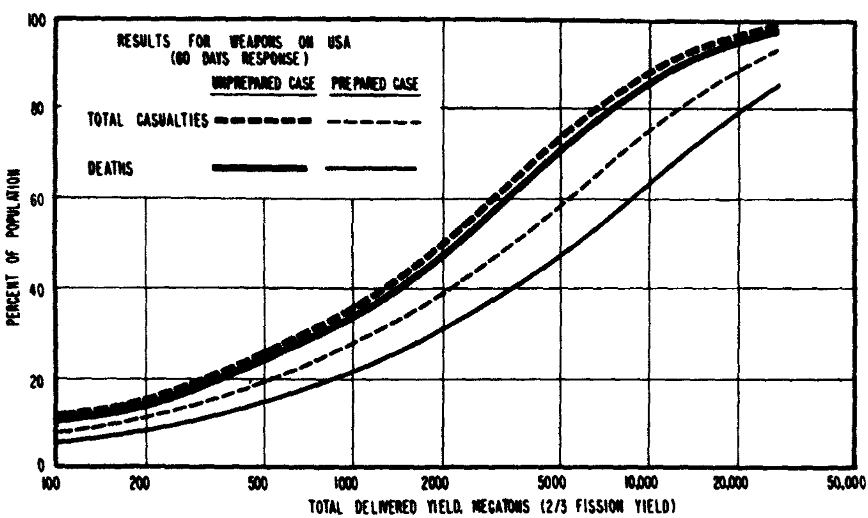

Fig 8 Population response in the United States resulting from 'optimal' attacks designed to *maximize deaths in a prepared population*

characteristic that weapons are dropped *only* in areas of high population density It is therefore quite an unlikely strategy since any strategic point in areas of low population density would be ignored if the procedure were followed to the letter On the other hand, some increase in casualties above the 'optimized' case would be obtained by using a small CEP and consistently targeting on cities or slightly upwind of them

The Proportional Drop

This case (Fig 9) is of interest because many installations of military importance are distributed in a way that resembles the population dis-

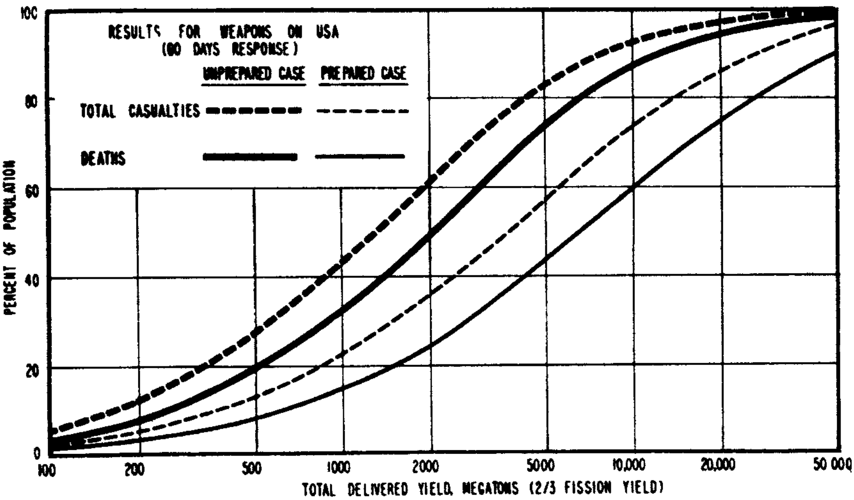

Fig 9 Population response in the United States resulting from attacks in which the delivered *yield density is proportional to the population density*

tribution Air attack on such facilities will probably produce results comparable to those obtained in this case

Uniform Drop

This case (Fig 10) is trivial and involves only the multiplication of the yield densities of Fig 5 by the total area of the country No claim is made for realism in this doctrine, but it is useful to see how sensitive the results are to extremes in targeting doctrine This corresponds to the extreme of not targeting at all It is not, however, a lower limit It is possible to minimize casualties by dropping all weapons at one point where the population density is minimum

It is interesting to note that for extremely large-yield campaigns the uniform drop becomes actually more efficient than the proportional This is because for high-yield attacks the proportional drop continues to use weapons where everyone is already dead

Of course, for all values of the total yield, an optimized program must remain above all other programs

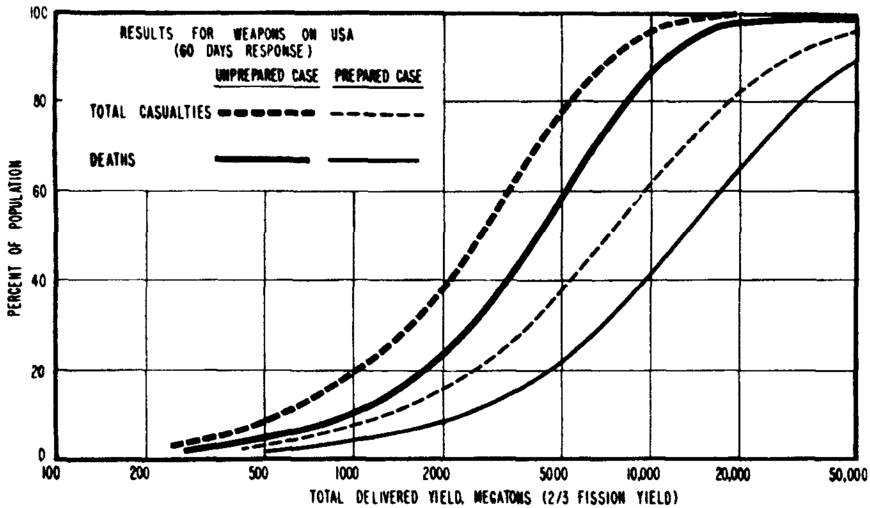

Fig 10 Population response in the United States resulting from attacks in which the *yield density is uniform over the entire country*

Air-Base Targeting

The curves calculated for weapons delivered to United States air bases in this example (Fig 11) assume the same allocation of yield to each SAC base, and one-half that allocation to other major military air bases

Casualties in this campaign turn out to be even lower than in the uniform drop case This reflects the fact that SAC bases are generally located in areas of low population density

The casualties and deaths (prepared and unprepared) obtained in a detailed map study of a similar campaign served as a check of the method The discrepancies between this method and the map study of the same campaign did not exceed 1 per cent of the population This agreement is no check of the parameters of either study since the same assumptions about shielding, biological response, and fallout area per weapon went into both studies It is, however, a check on the random drop model in the various sub-areas

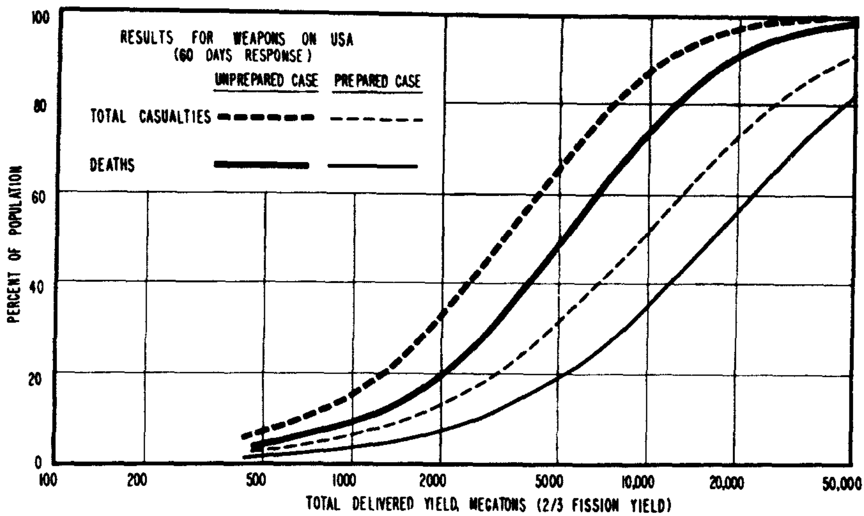

Fig 11 Population response in the United States resulting from attacks in which the *yield density is proportional to the distribution of military airbases*

COMPARISON WITH U S S R

THE CURVES in Figs 12 and 13 illustrate the sensitivity of the results to different target complexes Figure 12 is a comparison of the curves for unprepared deaths in the U S A for the four targeting doctrines discussed previously Figure 13 shows the result of the same type of calculations using the same shielding factors in the U S S R The somewhat smaller percentage fatalities for a given yield in the U S S R reflects both the larger population and the lower population density The effect of the vast empty area of Siberia is of course particularly evident in the results of the uniform drop For this calculation the U S S R was broken into twenty-six roughly homogeneous sub-areas, each composed of one to eighteen oblasts and containing 5 to 20 million people

APPLICABILITY OF THE FORMULAS

THE METHOD presented here provides a comparatively easy and versatile tool for rapidly estimating the effects of fallout in large campaigns It has the added advantage that it is based upon analytic formulas, so that any future changes in the knowledge of radiation distributions, biological response, or population shielding factors, can be easily incorporated

It has been our experience, on several occasions, that when a new targeting doctrine was presented it was possible to prepare a new set of results

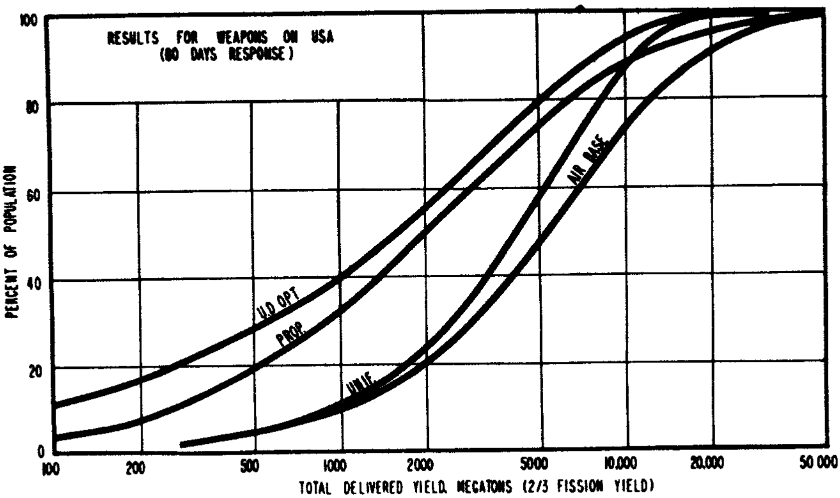

Fig 12 Comparison of deaths in an 'unprepared' population in the United States for different targeting doctrines *U D OPT*, 'optimal' distribution of yield-density to maximize deaths in the unprepared population, *PROP*, yield density proportional to population density, *UNIF*, yield density uniform in entire country, *AIR BASE*, yield density proportional to the distribution of military airbases

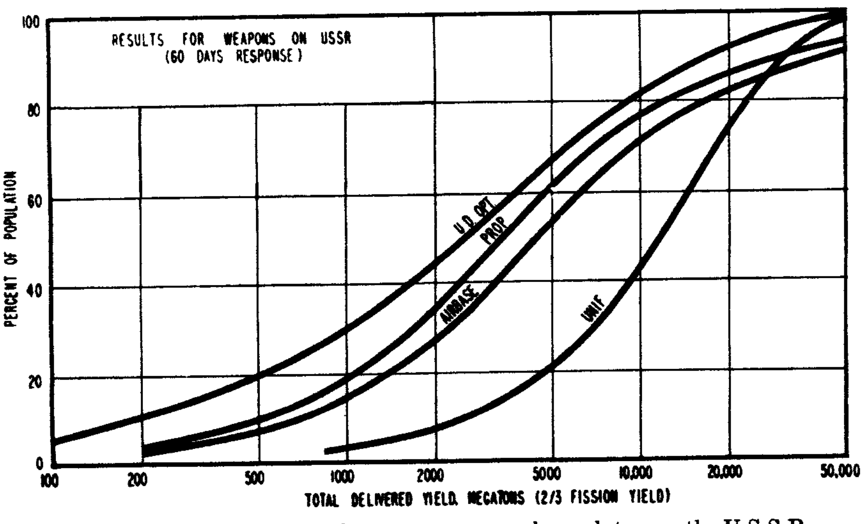

Fig 13 Comparison of deaths in an unprepared population in the U S S R for the same targeting doctrines shown in Fig 12

in a matter of several hours—not merely for one intensity of attack but for all force levels

It is, however, *essential* that certain precautions be observed by anyone attempting to apply the methods and results of this memorandum

1. The methods should only be applied to *large* campaigns (against large areas of the size, say, of the U S A , U S S R , Europe, etc , and total yields in excess of several hundreds of megatons) Only for such campaigns do the statistics allow fairly reliable conclusions without excessive variability, while at the same time minimizing the errors due to edge effects at the boundary of the area under attack

2. Strictly speaking, this calculation only accounts for *total radiation casualties at about 60 days* In the model there are *no blast effects whatsoever* However, almost all actual blast casualties will occur in regions of very high radioactive contamination, and will hence be counted as radiation casualties by this procedure The results of the method therefore closely approximate the expected *total casualties from all causes* Finally, it must be pointed out that the total casualties at 60 days may not be indicative of the ultimate casualties Such delayed effects as the disorganization of society, disruption of communications, extinction of livestock, genetic damage, and the slow development of radiation poisoning from the ingestion of radioactive materials may significantly increase the ultimate toll

3. The technique described here is based on the concept of a random drop of weapons It is therefore not appropriate to use it where there is a strong correlation between aiming points and population centers However, unless the correlation is very strong the error that results from doing so is surprisingly small In one case, which was considered where one 20 MT weapon was assigned to each of the first N largest cities of the U S A , the discrepancy was not great for the yields in excess of 1000 megatons However, for such campaigns it is much better procedure to calculate the expected casualties in the targeted cities separately These casualties are then removed from the population and the fraction of casualties expected in the remaining population can then be computed as before

It should be emphasized that the procedure cannot be used to predict casualties in any individual city The area casualties predicted are 'expected' values on the basis of a random distribution of wind On the other hand, it is impossible to predict the spatial distribution of fallout by any known method without accurate weather data at all altitudes It seems likely that a statistical treatment is all that can be achieved by any method when actual weather conditions are not known

4. The estimated effects are sensitive to the assumed biological response and shielding factors The formulas should not be used without an investigation to determine appropriate shielding factors

The response used here for the unprepared case corresponds roughly to a median, residual number of 0 40 where two-thirds of the residual numbers lie between 0 60 and 0 25, in the prepared case, the mean residual number used was about 0 11 and two-thirds of the residual numbers lie between 0 30 and 0 04 For the purpose of the original calculation, it was assumed that all fallout reached the ground at H_r+6 hours after burst The response was converted to a response in terms of the integrated 24-hr dose using the same assumption In order to use a different set of assumptions it is only necessary to compute the population response as a function of the integrated 24-hr dose for a series of unshielded dose levels The computed response is then fitted to the lognormal response curve as was done in this case

On the other hand, the shielding factors can be altered in rough computation simply by selecting different logarithmic means and variances It is worth remembering that, in order to achieve large average shielding factors, personnel must remain in the shield a high percentage of the time For instance, to get an average effective residual number of 0 01 a person in a perfect shelter could emerge from it only 1 per cent of the time Because this method is based on analytic formulas, in which the population response is represented, it is a simple matter to change population response to encompass new data

All other sources of error in the present approach are swamped by the uncertainty of these population-shielding parameters, an uncertainty which exists to the same degree in all fallout effects studies Therefore, we feel that this method, with its considerable advantages in simplicity and speed of application, can provide answers that are as reliable as the more detailed studies for the large area campaigns to which it is applicable

5. It was assumed in this treatment that all weapons were *surface burst* If it is desired to apply the method to campaigns that have air bursts, these *must not be counted* for these local fallout estimates Note that in this case, the remarks of 2 to the effect that the method also produces a fairly good estimate of total casualties (including blast) are *no longer applicable* The blast and other direct effects of the air-burst weapons must be treated separately

6 When applying the method to a campaign in which the weapons are not distributed uniformly over the entire area of attack, the area must be partitioned into sub-areas that have roughly homogeneous weapon distributions However, these sub-areas should still be sufficiently large that several weapons are targeted in each, and several times larger than the lethal area of a typical fallout pattern

7. In this paper, all weapon yields are assumed to have a two-thirds ratio of fission yield to nominal yield For other ratios, multiply the fission yield by $3/2$ to express the yield in our units

8. All weapons are presumed to be delivered within a time interval not longer than necessary so that they can be considered as simultaneously delivered, i e , so that there are no significant biological recovery effects during the total duration of the campaign (Probably two or three days suffices)