MEMORANDUM

To: Rainer Weiss

From: Peter Fisher

Subject: Collision Rate Between Primordial Black Holes and Sun Sized Stars - Estimates for the Milky Way

Date: January 22, 2017

The following estimate assumes primordial black holes (PBHs) for all of dark matter. The PBHs travel with $\beta = 0.001$, as is typical in CDM models. The calculation assumes the galaxy is a cylinder of 50 kpc radius and 1 kpc thickness uniformly filled with stars the size of our Sun and PBHs all of the same mass.

1. (PBH) mass range: $10^{12}\text{kg} - 10^{23}\text{kg}$. Lower limit comes from the need for the PBH to survive for 14 Gy against Hawking radiation, upper limit from weak lensing results. For scale, the mass of the Moon is $7 \times 10^{22}\text{kg}$.

2. Dark matter density at our location in the Milky Way: $\rho_{DM} = 0.5\text{GeV/cm}^3 = 9\times 10^{-22}\text{kg/m}^3$. The number density is
$$n_{PBH}(M) = \frac{5 \times 10^{-34}}{\text{m}^3}\left(\frac{10^{12}\text{kg}}{M}\right)$$

3. Cross section for a PBH to interact significantly or "hit" a Sun sized star: assume the PBH distance of closest approach is one solar radius $R_\odot = 7 \times 10^5\text{km}$. The impact parameter (see attached memo) $b = \sqrt{r_s R_\odot}/\beta$, $\beta \sim 0.001 = 10^{12}\text{m}$. Then the cross section is $\sigma = \pi r_s R_\odot/\beta^2 = 7 \times 10^{24}\text{m}^2$.

4. Stars in our part of the Milky Way are about 2 ly apart, so the stellar density is $n_{stars} = 1/(2\text{ly})^3 = 10^{-49}/\text{m}^3$.

5. Interaction depth $l = 1/n_{stars}\sigma = 10^{24}\text{m} = 10^8\text{ly}$. For a given PBH, the time between collisions with a star is $t = l/\beta c = 10^{11}\text{y}$.

6. Volume of the Milky Way: $V_{MW} = \pi R_{MW} T_{MW} = 1 \times 10^{62}/\text{m}^3$. The total number of PBHs is
$$N(M) = V_{MW} n_{PBH} = 2 \times 10^{29}\left(\frac{10^{12}\text{kg}}{M}\right).$$

7. The rate at which PBHs hit stars in the Milky Way is
$$R = \frac{N}{t} = 2 \times 10^{18}/\text{y}\left(\frac{10^{12}\text{kg}}{M}\right).$$

8. A detector with a range of 0.03 ly would make one detection per year. A detector with a 1 ly range, would make 7,000 detections per year.

9. The duration of the encounter is roughly the time it takes the PBH to significantly change direction, $t_{hit} = \pi R_{\odot}/\beta c = 7 \times 10^3$s or $\nu \sim 0.14$mHz.

10. If a PBH encounters a star, it will scoop up matter and lose energy. To compute the cross-section, use the total energy:

$$E_{tot} = E_{\infty} + V(r) = \gamma mc^2 - \frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{c^2mr^3} \tag{1}$$

The condition for capture is $V(r) > E_{\infty}$,

$$\frac{V(r)}{mc^2} = -\frac{r_s}{2r} + \frac{\beta^2 b^2}{2r^2}\left(1 - \frac{r_s}{r}\right) > \frac{E_{\infty}}{mc^2} = \gamma^2. \tag{2}$$

Solving numerically gives $b = 2,820 r_s$ giving $\sigma = 5 \times 10^{-23}\mathrm{m}^2$, Fig. 1. The average density of the Sun is $\rho_{astrosun} = 1,392\mathrm{kg/m}^3$. Taking a "typical" path as $R_{\odot}$, a PBH of 10^{12} kg collects

$$dM \sim \sigma \rho_{\odot} R_{\odot} = 5 \times 10^{-11}\mathrm{kg}. \tag{3}$$

Such a PBH will pick up 2×10^{-20}kg from the ISM in the age of the universe, assuming 1 proton per cc.

Carrying out the same calculation for 10^{-23}kg gives $dM = 5 \times 10^{11}$kg per stellar transit and 10^2kg from the ISM.

In neither case are the mass changes sufficient to slow down the PBH.

11. For what β will a PBH pick up sufficient stellar mass to slow down? From Eq. **??**, the scaling is for a 10^{23} kg PBH is

$$dM \sim 5 \times 10^{11}\mathrm{kg} \times \left(\frac{0.001}{\beta}\right)^2. \tag{4}$$

Setting $dM = 0.1 M_{\odot}$ gives $\beta \sim 10^{-7}$.

12. The strain produced from gravitational radiation from the PBH in a stellar encounter is given by,

$$h = \frac{GM}{Lc^2}\beta^2,$$

where h is the strain, L is the distance from the PBH, and β is the velocity associated with the acceleration. For $M = 10^{12}$kg, $h = 10^{-37}$.

Assuming a few hour pulse length, this event would be in the detection frequency band of LISA which will at best have strain noise at periods around a few hours of $h \sim 10^{-20}$.

Take the largest PBH $M = 10^{23}$kg, the strain goes to $h \sim 10^{-26}$. Then with a temporally unresolved group of bursts N in a bend time, you might get a $h_{rms} \sim h\sqrt{N}$. If there are 10^4 events per bend time you could get $h_{rms} \sim 10^{-24}$. This might be in the sensitivity range of the "big bang observer" project which if it happens at all will be after LISA.

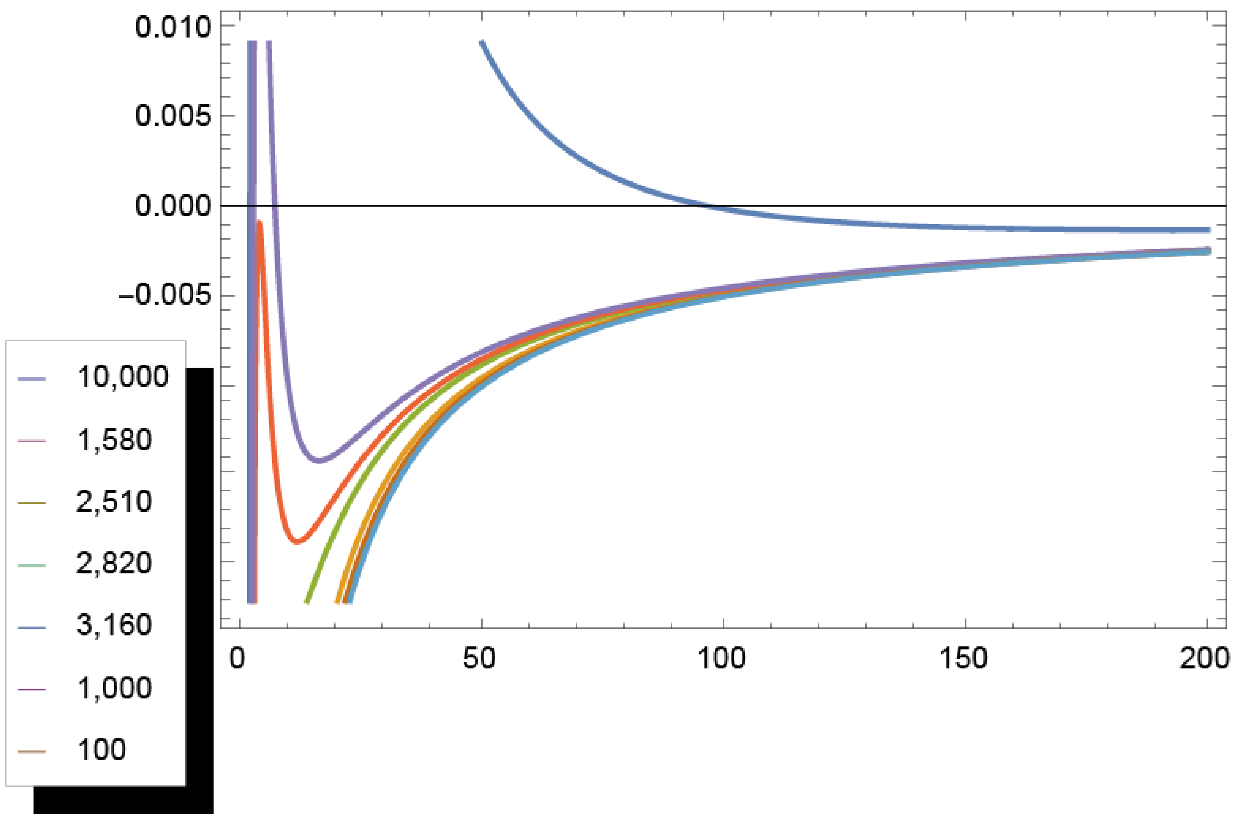

Figure 1: $V(r)$ for different values of b. The condition in Eq. 2 is met for $b = 2,820r_s$, shown as the red curve.

13. The cross section for a PBH to hit a star and the PBH number density depend on β, which follows a Maxwell distribution. The cross section (Item 3) scales as $\sigma \propto 1/\beta^2$, so the interaction depth (Item 4) scales as $l \propto \beta^2$ and the mean time between interactions $t = l/\beta c \propto \beta$ and the collision rate of a single PBH is $1/t = \beta c n_s \sigma \propto 1/\beta$.

14. The PBH velocity distribution follows the Maxwell distribution with a dispersion of $\beta = 0.001$. Below $\beta \sim 10^{-4}$, a good approximation is,

$$\frac{d^2N}{d\beta dV} = n_{PBH} 10^{8.90} \beta^3$$

and

$$\begin{aligned} \frac{d^2 N_{coll}}{d\beta dt} &= \frac{V_{mw} n_{PBH} 10^{8.90} \beta^3}{t_{univ}} \begin{pmatrix} \text{Collision rate} \\ \text{per PBH} \end{pmatrix} & (5) \\ &\propto \frac{\beta^3}{\beta} \propto \beta^2 & (6) \end{aligned}$$

The number of PBH collisions with $\beta \sim 10^{-7}$ is then 5×10^{-12}/s in the Milky Way.