Notes on computing the lifetime for $0\nu\beta^{\pm}\beta^{\pm}$ and $0\nu\beta^{\pm}\beta^{\mp}$

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Abstract

ALLOWED DOUBLE BETA DECAY

Matrix Element

$$\mathcal{M} = H^{\mu\nu} L^{\mu\nu} \tag{1}$$

$$\tag{2}$$

Leptonic part

$$\begin{aligned} L^{\mu\nu} &= \overline{v}\left(p_+\right)_R \gamma_\nu \left(\not{q} + m_\nu\right) \gamma^\mu u\left(p_-\right)_L && (3)\\ &= \overline{v}\left(p_+\right) \frac{1}{2}\left(1+\gamma_5\right) \gamma_\nu \left(\not{q} + m_\nu\right) \gamma^\mu \frac{1}{2}\left(1-\gamma_5\right) u\left(p_-\right) && (4)\\ &= \overline{v}\left(p_+\right) \gamma_\nu \frac{1}{2}\left(1-\gamma_5\right) \left(\not{q} + m_\nu\right) \frac{1}{2}\left(1+\gamma_5\right) \gamma^\mu u\left(p_-\right) && (5)\\ &= \overline{v}\left(p_+\right) \gamma_\nu \frac{1}{2}\left(1-\gamma_5\right) \not{q} \gamma^\mu u\left(p_-\right) && (6) \end{aligned}$$

$$\begin{aligned} \overline{L}^{\sigma\rho} L^{\mu\nu} &= \overline{u}\left(p_-\right) \gamma^\rho \frac{1}{2}\left(1+\gamma_5\right) \not{q} \gamma^\sigma v\left(p_+\right) \overline{v}\left(p_+\right) \gamma^\nu \not{q} \frac{1}{2}\left(1+\gamma_5\right) \gamma^\mu u\left(p_-\right) && (7)\\ &= \left(\frac{\not{p}_- + m_e}{2m_e}\right) \gamma^\rho \frac{1}{2}\left(1+\gamma_5\right) \not{q} \gamma^\sigma \left(\frac{-\not{p}_+ + m_e}{2m_e}\right) \gamma^\nu \not{q} \frac{1}{2}\left(1+\gamma_5\right) \gamma^\mu. && (8) \end{aligned}$$

There are size gamma matrices outside the energy projectors, so only the combinations with an even number of gamma matrices will not be zero.

$$\begin{aligned} \overline{L}^{\sigma\rho} L^{\mu\nu} = &\left(\frac{\not{p}_-}{2m_e}\right) \gamma^\rho \not{q} \gamma^\sigma \left(\frac{-\not{p}_+}{2m_e}\right) \gamma^\nu \not{q} \frac{1}{2}\left(1+\gamma_5\right) \gamma^\mu && (9)\\ &+ \left(\frac{\not{p}_-}{2m_e}\right) \gamma^\rho \gamma_5 \not{q} \gamma^\sigma \left(\frac{-\not{p}_+}{2m_e}\right) \gamma^\nu \not{q} \frac{1}{2}\left(1+\gamma_5\right) \gamma^\mu && (10)\\ &+ \left(\frac{m_e}{2m_e}\right) \gamma^\rho \not{q} \gamma^\sigma \left(\frac{m_e}{2m_e}\right) \gamma^\nu \not{q} \frac{1}{2}\left(1+\gamma_5\right) \gamma^\mu && (11)\\ &+ \left(\frac{m_e}{2m_e}\right) \gamma^\rho \gamma_5 \not{q} \gamma^\sigma \left(\frac{m_e}{2m_e}\right) \gamma^\nu \not{q} \frac{1}{2}\left(1+\gamma_5\right) \gamma^\mu && (12) \end{aligned}$$

Useful Relations

Spinors

$$\chi^{+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \chi^{-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{13}$$

Wave functions

$$\psi_n = \sqrt{\frac{E + mc^2}{2mc^2}} u_n e^{-(iEt - i\vec{p}\cdot\vec{x})} \tag{14}$$

$$u_1 = \begin{pmatrix} \chi^{+} \\ \frac{pc}{E+mc^2}\chi^{+} \end{pmatrix} u_2 = \begin{pmatrix} \chi^{-} \\ -\frac{pc}{E+mc^2}\chi^{-} \end{pmatrix} \tag{15}$$

$$v_1 = \begin{pmatrix} \frac{pc}{E+mc^2}\chi^{+} \\ \chi^{+} \end{pmatrix} v_2 = \begin{pmatrix} -\frac{pc}{E+mc^2}\chi^{-} \\ \chi^{-} \end{pmatrix} \tag{16}$$

$$\tag{17}$$

Charge Conjugation

$$C = i\gamma^2\gamma^o = \begin{pmatrix} 0 & -i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \tag{18}$$

$$Cu_1 = -v_2 Cu_2 = \gamma^o v_1 \tag{19}$$

$$Cv_1 = -\gamma^o u_2 Cv_2 = -u_1 \tag{20}$$

Chiral States

$$Cu_{1,L} = \gamma^o v_{1,R} \tag{21}$$

$$u_L = \frac{1}{2}\left(1-\gamma_5\right)u \tag{22}$$
$$\overline{u}_L = u_L^{\dagger}\gamma^o \tag{23}$$
$$= \left[\frac{1}{2}\left(1-\gamma_5 u\right)\right]^{\dagger}\gamma^o \tag{24}$$
$$= u^{\dagger}\frac{1}{2}\left(1-\gamma_5\right)\gamma^o \tag{25}$$
$$= u^{\dagger}\gamma^o\left(1+\gamma_5\right) \tag{26}$$
$$= \overline{u}\frac{1}{2}\left(1+\gamma_5\right) \tag{27}$$

Consider $W^- \to e^-\overline{\nu}_e$. The leptonic current is:

$$J_l^{\mu} = \overline{v}_{\overline{\nu}_e,R}\gamma^{\mu}u_{e^-,L} \tag{28}$$
$$= \overline{v}_{\overline{\nu}_e}\gamma^{\mu}\frac{1}{2}\left(1-\gamma_5\right)u_{e^-} \tag{29}$$
$$= \overline{v}_{\overline{\nu}_e}\frac{1}{2}\left(1+\gamma_5\right)\gamma^{\mu}u_{e^-}. \tag{30}$$

So,

$$\overline{v}_R = \frac{1}{2}\left(1+\gamma_5\right)\overline{v}. \tag{31}$$

Next, consider $W^+ \to e^+\nu_e$, whose current is,

$$J_l^{\mu} = \overline{u}_{\nu_e,L}\gamma^{\mu}v_{e^+,R} \tag{32}$$
$$= \overline{u}_{\nu_e,L}\frac{1}{2}\left(1+\gamma_5\right)\gamma^{\mu}v_{e^+,R} \tag{33}$$
$$= \overline{u}_{\nu_e,L}\gamma^{\mu}\frac{1}{2}\left(1-\gamma_5\right)v_{e^+} \tag{34}$$

and,

$$v_R = \frac{1}{2}\left(1-\gamma_5\right)v \tag{35}$$

Chiral states and energy projectors:

$$\frac{1}{2}\left(1-\gamma_5\right)\left(\not{p}+m\right)\frac{1}{2}\left(1+\gamma_5\right) = \frac{1}{2}\left(1-\gamma_5\right)\not{p}$$
$$\frac{1}{2}\left(1-\gamma_5\right)\left(\not{p}+m\right)\frac{1}{2}\left(1-\gamma_5\right) = \frac{1}{2}\left(1-\gamma_5\right)m$$