physics@mit

MEMORANDUM

To: Doug Eardley, Graham Candler

From: Peter Fisher

Subject: X-band range and range-rate with a Kalman filter

Date: July 17, 2018

In a previous memo, the use of X or K band radar was considered to compute range and range-rate for a target. This note describes the next step oin the process - the use of a filtering algorithm to make an optimal measurement of t_{go} and its uncertainty in real time.

The system describes a head-on engagement starting 100 km apart with the vehicles approaching at 10 km/s. At $10 < d < 80$ km, the approaching vehicle accelerates at $30g$ for 0.1 s, gaining Δv=30 m/s velocity and continues at at constant velocity.

The state vector is $\vec{x} = (r, \dot{r})$ and the measurement vector is $\vec{z} = (\Delta t, \Delta\nu)$. $\vec{z} = H\vec{x}$ and the measurement matrix is,

$$H = \begin{pmatrix} \frac{1}{c} & 0 \\ 0 & \frac{\Delta\nu}{2c}. \end{pmatrix} \tag{1}$$

At each step, the filter computes the uncertainties on the distance and velocity uncertainties from,

$$\sigma_R = \frac{cB_{pulse}}{4\tau_{pulse}\sqrt{SNR}\sqrt{N_{sample}}} \tag{2}$$

$$\sigma_V = \frac{\lambda}{4\tau_{pulse}\sqrt{SNR}\sqrt{N_{sample}}} \tag{3}$$

with $\lambda = 15\text{cm}$, $B_{pulse} = 1\text{MHz}$, $\tau_{pulse} = 1\mu\text{s}$, $N_{sample} = 500$, and $SNR = 10\,(100\text{km}/R)^4$. See the previous memo [1] for the origin of these values from X band radar.

The filter predicts the next value from the previous measurements, updates using the next measurement weighted by the Kalman gain calculated at each step, and recomputes the covariance matrix, Fig. 1. For each update, the filter adds a "control" signal if the predicted residual on the k^{th} frequency measurement $y_{k,\Delta\nu} > 3\sigma_{k,\Delta\nu}$, where $\sigma_{k,\Delta\nu}$ is computed from $\sigma_{k,V}$. The control signal vector and variance are

$$\vec{u} = \begin{pmatrix} 0 \\ -\frac{x_{k|k,2} - x_{k-1|k-1,2}}{dt} \end{pmatrix} \tag{4}$$

$$Q = \begin{pmatrix} \frac{dt^4}{4} & \frac{dt^3}{2} \\ \frac{dt^3}{2} & dt^2 \end{pmatrix} \times \sqrt{2}\left(\frac{\sigma_V}{dt}\right)^2 \tag{5}$$

The control signal has the effect of nulling out the residual state vector from the maneuver and the "noise" matrix increases the covariance and Kalman gain, increasing the impact of the new measurements in the current and future state vectors. Fig. 1 shows these effects for maneuvers taking place at 10, 20, 30, and 40 km. As the distance of the maneuver increases, the covariance, Kalman gain, and residuals all increase dramatically. At 40 km, the change in the residual is less than 3 σ and the filter "misses" the maneuver. At 40 km, $SNR = 390$, and this represents a minimum noise level to detect a 30 g maneuver using the method described above.

When the filter responds to a maneuver, the Kalman gain increases significantly because the covariance matrix P has had the process noise of the response added to it. For both K_{XX} and P_{XX}, the recovery lasts about 0.2 s and during this time, the distance measurement will have larger predicted residuals than before the maneuver. In contrast, K_{YY} and P_{YY} return to their pre-maneuver values just a few samples after the maneuver ends.

An the current estimate of the uncertainty for t_{go} is,

$$\sigma_{t_{go}} = \left(\frac{\partial t_{go}}{\partial R}\right)^2 \sigma_R^2 + 2\left(\frac{\partial t_{go}}{\partial R}\right)\left(\frac{\partial t_{go}}{\partial V}\right)\sigma_{RV}^2 + \left(\frac{\partial t_{go}}{\partial V}\right)^2 \sigma_V^2 \tag{6}$$

$$= \vec{s}_k P_{k|k} \vec{s}_k \tag{7}$$

$$\vec{s}_k = \begin{pmatrix} \frac{1}{V} \\ -\frac{R}{V^2}. \end{pmatrix} \tag{8}$$

Fig. 2 shows a comparison of the predicted t_{go} with the residual between the current predicted value and the true value. The uncertainty predicted by the filter is about 1 ms during the maneuver and drops exponentially once the maneuver is complete. If better accuracy is needed, the Kalman update time could be lowered from 10 ms to 3 ms.

This study shows the X band radar described in [1] could be used with a Kalman filter operating with 10 ms updates to provide t_{go} with 1 ms accuracy for maneuvers at 10's if g. The filter's performance could be improved by lowering the update time or the trigger frequency residual. The performance of a K-band radar would be better if used above 50 km.

This same analysis could be applied to LIDAR.

References

[1] Fisher, P., "An X and K-band radar for ranging during a terminal engagement", July 16, 2018.

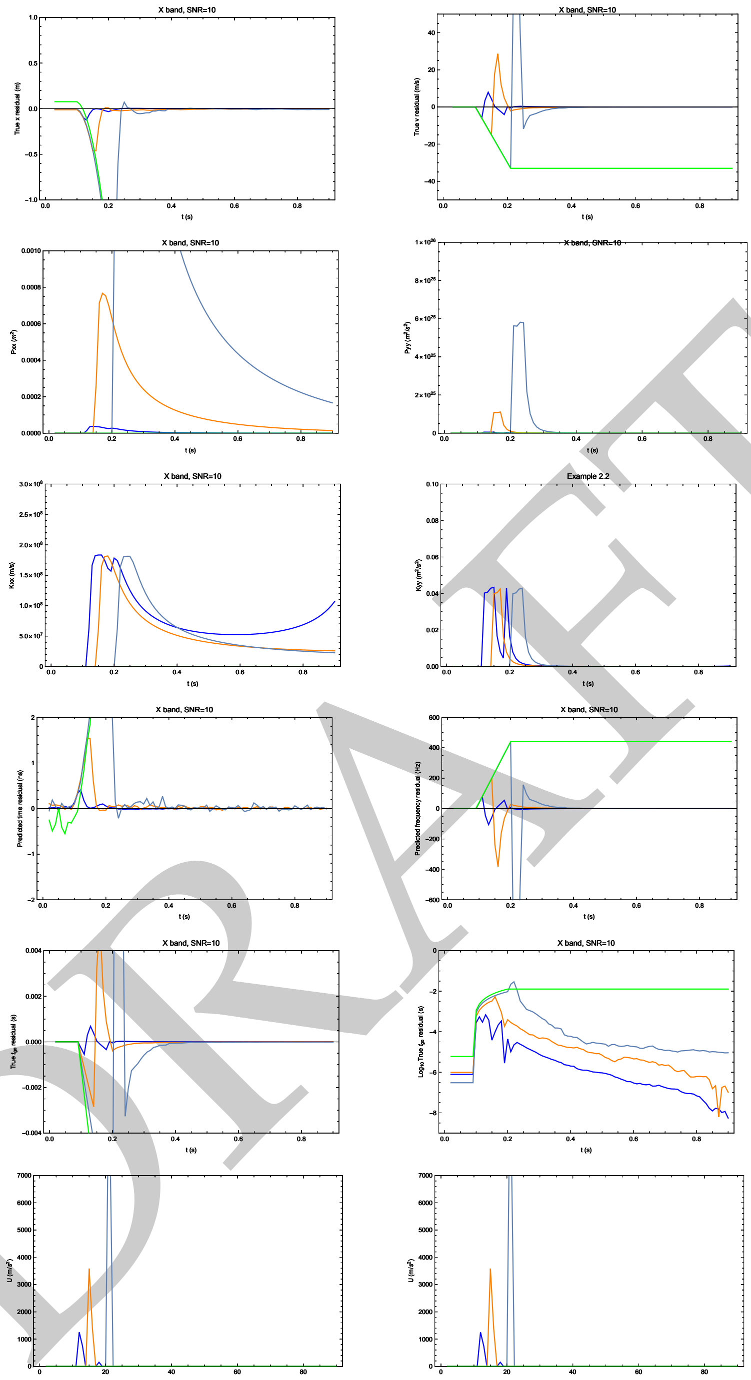

Figure 1: Plots of time, frequency, distance, and velocity residuals and principal components of K and P as functions of time. Each curve represents the response to a maneuver at a different distance: the blue curve is 10 km, orange curve is 20 km, teal curve is 30 km, and green curve is 40 km. The maneuver always takes place 1 s after the start of the calculation and lasts 1 s. The 40 km green curve is frequently off scale because residual is below the 3 σ requirement for a control response. The K_{XX} and P_{XX} refer the the distance Kalman gain and covariance, K_{YY} and P_{YY} refer to the velocity.

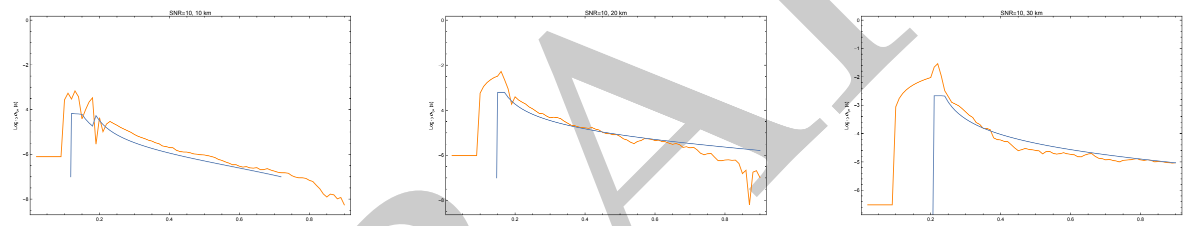

Figure 2: Teal curve shows the filter's prediction of $\sigma_{t_{go}}$ and the orange curve shows the true residual.