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MEMORANDUM

To: Paul Dimotakis

From: Peter Fisher

Subject: Studies of rockets related to BPI

Date: July 19, 2018

This note describes studies of various rocket configuration for the BPI study. First order numerical integration is used to integrate the Newton's laws and includes gravity, drag, and staging. The program accommodates three stage rockets with a different mass and burn time for each stage. The $\dot{m}$ is constant and the forces acting on the rocket are,

$$\vec{F}_D = -\frac{1}{2}C_D\rho A v^2 \hat{a} \quad (1)$$

$$\vec{F}_g = -mg\hat{z} \quad (2)$$

$$\vec{F}_T = \dot{m} I_{sp}(z)\, g \quad (3)$$

where A is the cross sectional area of the rocket, C_D is the drag coefficient, and I_{sp} is the specific impulse.

The specific impulse is calculated as a function of altitude to take into pressure outside the nozzle,

$$I_{sp}(z) = I_{sp,vac} + \frac{P(z)}{P(0)}\left(I_{sp,sl} - I_{sp,vac}\right)$$

where sl refers to sea level and vac refers to vacuum. Table 1 gives the specific impulses used.

Fuel	$I_{sp,vac}$ (s)	$I_{sp,sl}$ (s)
HTPB	274	245
UDMH	333	285

Table 1: Specific impulses used in this calculation. Values for HTPB (solid) are from [1] and UDMH are from [**?**].

The program carries out the calculation in two dimensions. The rocket fires vertically and proceeds straight-up until a specified altitude, at which point a fraction, typically 2%, of the thrust is diverted normal to the rocket body to pitch the rocket to a specified angle, at which point the rotation is stopped. The moment of inertial is computed for the length and mass of the missile at the time of pitch over. The balance of the thrust and drag are directed along the rocket axis and the gravitational force acts in the $-\hat{z}$m direction. Table 2 gives the parameters of the missiles studied and Table 4 gives the flight variables adjusted for each calculation.

Parameter	Unit	1800-65-30a	Hwasong-15
Stage 1 mass	kg	1170	53,700
Stage 2 mass	kg	487	17,897
Stage 3 mass	kg	142.9	-
Payload mass	kg	30	500
Stage 1 length	m	4.3	13.5
Stage 2 length	m	1.98	4.5
Stage 3 length	m	0.5	-
Rocket diameter	m	0.5	2.4
Stage 1 propellent fraction	%	89.8	88
Stage 2 propellant fraction	%	89.8	88
Stage 3 propellant fraction	%	79.0	88
Stage 1 burn time	s	21.2	150
Stage 2 burn time	s	21.2	100
Stage 3 burn time	s	207.6	-
Stage 1 fuel		HTPB	UDMH
Stage 2 fuel		HTPB	UDMH
Stage 3 fuel		UDMH	-
C_D		0.2	0.2

Table 2: Representative parameters used in this study.

Variable	Units	Typical range of values
Pitch over angle θ_l	deg.	-2 - 2
Rotation thrust divert	%	2-3
Pitchover altitude	km	15.2-20

Table 3: Flight variables.

A previous note [**?**] gave the burn out velocity for the Hwasong-15 test flight in November 2017 that showed a burn out velocity of 7.2 km/s, imply the rocket could reach large part of the United States. The numbers given in Table 2 above were used in the calculation to give a notional trajectory shown in Fig. 1. This trajectory represents a guess - the details of the rocket pitch-over to the 22° will determine the range during boost and this is unknown at this time. This model uses 50 km for the pitchover altitude and at the burn out of the second stage, the rocket is 481 km down-range, at an altitude of 327 km, and traveling at 7.1 km/s.

A simple optimization of the flight parameters for version 8000-65-30a shows a target (a Hwasong-15) with the flight parameters given above) can be intercepted in a tail-chase starting from a point 485 km behind the target launch point, Fig. 2. A boost phase intercept requires catching the target before the burn out of the final stage and this requires the interceptor to pitch-over to zero degrees soon enough to acquire enough horizontal velocity to catch the target while still having enough vertical velocity to reach the target's burn out altitude. Ideally, interception should take place with the interceptor having zero vertical velocity. Pitchover should take place above as much of the Earth's atmosphere as possible and after the burn out of the first stage, in order that the

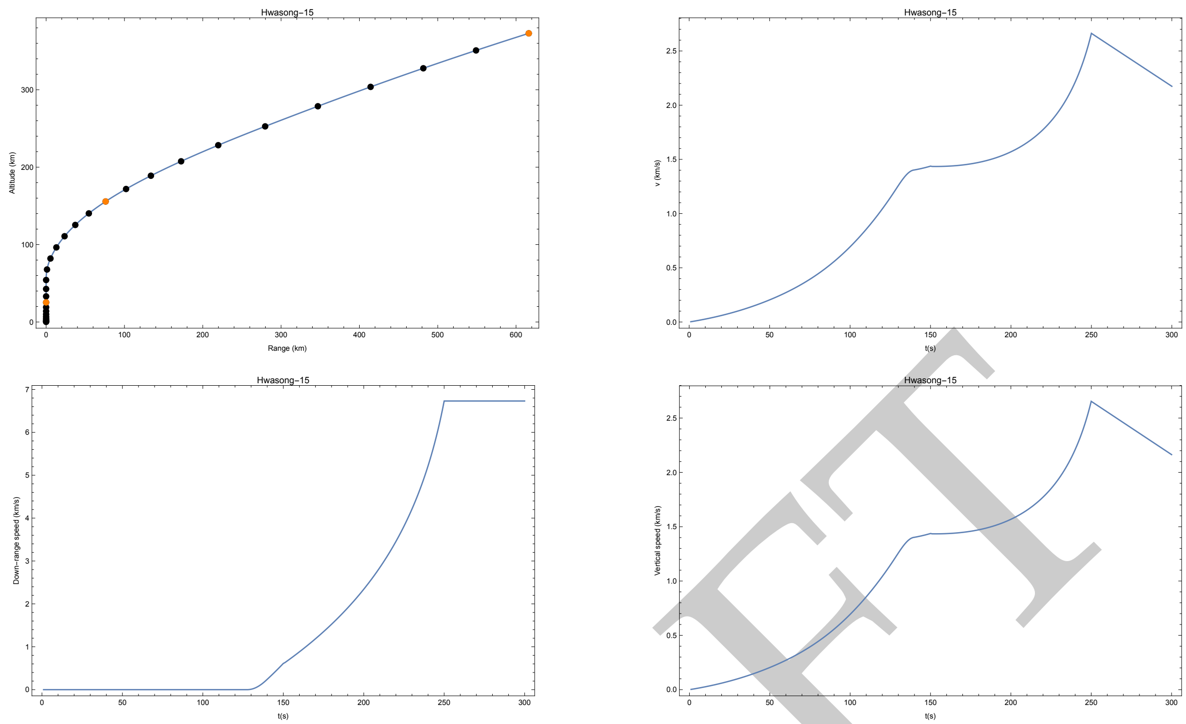

Figure 1: Trajectory, total speed, horizontal, and vertical speeds for Hwasong-15. In the upper left panel, the black dots denote 10 s time intervals, the orange dots denote 100 s time intervals.

Stage	t (s)	v (km/s)	Δ_V (km/s)	ΔV_T (km/s)	ΔV_g (km/s)	ΔV_d (km/s)
1	21.2	1,588	1,588	2,220.2	-208.5	-426.8
2	21.2	4,288	2,700	3,160	-208.0	-261.8
3	250	7,338	3,050	5,082	-2,036	-1.1

Table 4: Velocity changes during of the interceptor. Each ΔV_T results from numerical integration during the flight and agrees with the calculation from Eq. **??** for the parameters in Table 2.

maneuver take place quickly.

The interception in the flight shown in Fig. 2 takes place 6 s after the burn out of the interceptors third stage. Table **??** shows some parameters of the flight, comparing with velocity changes for each stage as computed by the Tsiolkovsky rocket equation,

$$\Delta V\,(t) = I_{sp} g \ln \frac{m_i}{m_i - \dot{m}t}.$$

Drag has a net effect of less than 0.5 km/s, mostly during the burn of the first stage. Gravity has a large effect, 2.5 km/s, that occurs during the 201.6 s burn of the third stage because $\Delta V_g = gt$. There is an advantage to pitching over as early as possible and intercepting the target earlier than burn-out.

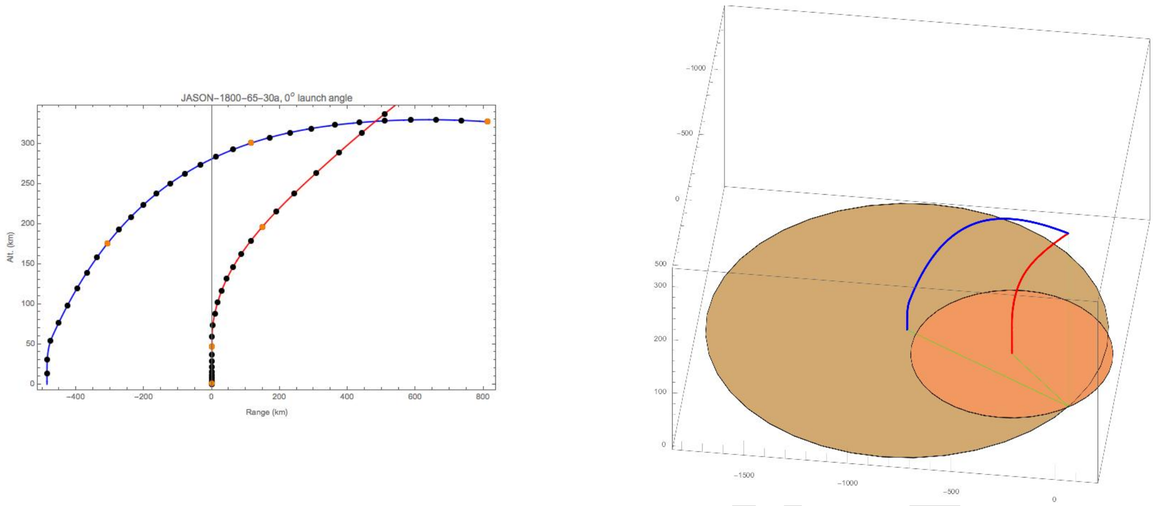

Figure 2: Interception trajectory. In the left panel, the interception occurs just at target burn out. The interceptor was launched 24 s after the target. The black dots mark every 10 s and orange ever 100 s since target launch. The right panel shows the same interception in which the target has launch on a heading toward Washington DC from Pyongsang and the interceptor has been launched from the eastern end of the DMZ.

References

[1] http://www.braeunig.us/space/propel.htm.

[2] Fisher, P, “Orbit calculations between two points on Earth”, memo from July 16, 2018.

[3] http://www.astronautix.com/n/n2o4udmh.html.