MEMORANDUM

To: ALCON

From: Peter Fisher

Subject: A'rate with radiative elastic backgrounds

Date: Nov. 7, 2017

The notational detector is a twin arm spectrometer, one for detecting electrons (the electron arm), one for detecting positrons (the positron arm). The arms are located on either side of the beam line at angles $\theta^+$and θ^-subtending solid angles $d\Omega^+$ and $d\Omega^-$, respectively.

Rate of radiative elastic scatters that trigger the electron arm:

$$\frac{dN^-_{el,\gamma}}{dt} = \frac{d\sigma_{el,\gamma}}{d\Omega} d\Omega_- I\rho t = \Sigma^-_{el,\gamma} I,$$

where I is the number of electrons per second on the target, ρ is the target number density, t is the target thickness, and

$$\Sigma^-_{el,\gamma} = \frac{d\sigma_{el,\gamma}}{d\Omega} d\Omega_- \rho t$$

is dimensionless number.

The signal rate from A' decay is,

$$\frac{dN^{+-}_{A'}}{dt} = \Sigma^{+-}_{A'} I,$$

where the superscript indicates both arms of the spectrometer trigger.

The QED rate is,

$$\frac{dN^{+-}}{dt} = \Sigma^{+-}_{QED} I.$$

Accidental coincidences consist of events with a radiative elastic scatter into the electron arm and a QED or A' event with a positron that triggers the positron arm and an electron the misses the electron arm. The coincidence time window between the two arms is Δ. The rate is then,

$$\begin{aligned}
\frac{dN^{+-}_{acc}}{dt} &= \frac{dN^-_{el,\gamma}}{dt} \left(\frac{dN^+_{QED}}{dt} - \frac{dN^{+-}_{QED}}{dt} \right) \Delta \frac{\Delta M}{M} \\
&= \Sigma^-_{el,\gamma} I \left(\Sigma^+_{QED} I - \Sigma^{+-}_{QED} I \right) \Delta \frac{\Delta M}{M} \\
&= \Sigma^-_{el,\gamma} I^2 \Delta \left(\Sigma^+_{QED} - \Sigma^{+-}_{QED} \right) \frac{\Delta M}{M}.
\end{aligned}$$

$\Delta M/M$ takes into account the mass resolution of the spectrometer. Assuming the QED and accidental backgrounds are roughly flat, we only need to account for the events in the A'mass bin.

The signal to background ratio in the A'mass bin for a running time τ is

$$\begin{aligned}\frac{N_{sig}}{\sqrt{N_{bkg}}} &= \frac{\Sigma_{A'}^{+-} I\tau}{\sqrt{\Sigma_{QED}^{+-}\frac{\Delta M}{M} I\tau + \Sigma_{el,\gamma}^{-} I^2\tau\Delta\left(\Sigma_{QED}^{+} - \Sigma_{QED}^{+-}\right)\frac{\Delta M}{M}}} \\ &= \sqrt{\frac{I\tau}{\frac{\Delta M}{M}}}\frac{\Sigma_{A'}^{+-}}{\sqrt{\Sigma_{QED}^{+-} + I\Delta\Sigma_{el,\gamma}^{-}\left(\Sigma_{QED}^{+} - \Sigma_{QED}^{+-}\right)}}\end{aligned}$$

$I\tau$ and $I\Delta$ are dimensionless numbers.

We can roughly relate the different dimensionless cross sections. Σ to be a total dimensionless differential cross section. Each arm of the spectrometer subtends a solid angle $d\Omega = g \sim 0.01$, so

$$\Sigma^{-} \sim \Sigma^{+} \sim g\Sigma.$$

and

$$\Sigma^{+-} \sim g^2\Sigma$$

The QED cross section is smaller by a factor of α from the radiative elastic cross section,

$$\Sigma_{QED} \sim \alpha\Sigma_{el,\gamma}.$$

Finally,

$$\Sigma_{A'} = \epsilon^2\Sigma_{QED}$$

and we are aiming for $\epsilon^2 \sim 10^{-7}$.

Now cast Eq. 1 in terms of $\Sigma_{el,\gamma}$,

$$\begin{aligned}\frac{N_{sig}}{\sqrt{N_{bkg}}} &= \sqrt{\frac{I\tau}{\frac{\Delta M}{M}}}\frac{g^2\epsilon^2\alpha\Sigma_{el,\gamma}}{\sqrt{g^2\alpha\Sigma_{el,\gamma} + I\Delta g\Sigma_{el,\gamma}\left(g\alpha\Sigma_{el,\gamma} - g^2\alpha\Sigma_{el,\gamma}\right)}} \\ &= \sqrt{\frac{I\tau}{\frac{\Delta M}{M}}}\frac{g\epsilon^2\sqrt{\alpha\Sigma_{el,\gamma}}}{\sqrt{1 + I\Delta\Sigma_{el,\gamma}\left(1-g\right)}} \\ &= \sqrt{\frac{I\tau}{\frac{\Delta M}{M}}}\frac{g\epsilon^2\sqrt{\alpha\Sigma_{el,\gamma}}}{\sqrt{1 + I\Delta\Sigma_{el,\gamma}}},\end{aligned}$$

since $g << 1$. The $\Delta M/M$ in eq. 1 factor reduces the error by a factor of 10 for the ^{8}Be measurement since $\Delta M \sim 200$keV and $M \sim 20$ MeV. If $I\Delta\Sigma_{el,\gamma} >> 1$, the signal to background ratio increases as $\sqrt{\tau}$, independently of I. This is just because the second term in the denominator comes from accidental coincidences that goes as I^2, cancelling the I in the numerator.

This says that $I_{max} \sim 1/\Delta\Sigma_{el,\gamma}$ gives the maximum current needed to carry out the measurement. Increasing the current beyond this value does not improve the measurement statistics at the signal rate is equal to the square root of the background rate for any $I > I_{max}$. For the radiative Rutherford cross section,

$$\begin{aligned}\frac{d\sigma}{d\Omega}_{el} &= \left[\frac{Z_1 Z_2 \alpha \hbar c}{4E\sin^2\theta/2}\right]^2 \\ \frac{d\sigma}{d\Omega}_{el,\gamma} &= \frac{d\sigma}{d\Omega}_{el}\frac{\alpha}{2k\left(2\pi\right)^3}k^2 d\Omega_k dk\left(\frac{\epsilon\cdot p_f}{k\cdot p_f}-\frac{\epsilon\cdot p_i}{k\cdot p_i}\right)^2\theta\left(E_i-m-k\right)\end{aligned}$$

where p_i, p_f, and E_i refer to the electron initial momentum, final momentum, and energy, k is the photon energy and ϵ the photon polarization. Using $\rho = 20\mathrm{g/cm}^3$, $t = 10\mu$, $E = 50$ MeV, $Z_1 = 1$, $Z_2 = 74$, $\theta = 45°$, and $\Delta = 10^{-8}$ s, gives an estimate of the maximum current. Estimating $d\sigma/d\Omega_{el,\gamma}$ from Eq. 1 is difficult without a full simulation– $d\sigma/d\Omega_{el} \sim 10^{-29}\mathrm{m}^2$ at $\theta = 45°$ and the in the extreme relativistic limit, the radiative part is a multiplicative factor of

$$\frac{2\alpha}{\pi}\ln\frac{k_{max}}{k_{min}}\left(\ln\frac{-q^2}{m^2}-1\right)\sim 0.014$$

for $k_{max}/k_{min} = 5$ and$-q^2 = 5$ MeV^2. This gives $\Sigma_{el,\gamma} \sim 10^{-7}$ and $I \sim 10^{12}e/s = 0.1$mA. An estimate of the signal to background ratio is,

$$\frac{N_{sig}}{\sqrt{N_{bkg}}} = \frac{4\times 10^{-6}}{\sqrt{s}}\sqrt{\tau}.$$

This implies accumulating enough statistics for a significant measurement would take 5 $\times 10^{10}$s or 1,700 y.