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MEMORANDUM

To: ALCON

From: Peter Fisher

Subject: "The Cause of the Formation of Meanders in the Courses of Rivers and the s-called Baer's Law", A. Einstein

Date: March 1, 2018

THE CAUSE OF THE FORMATION OF MEANDERS IN THE COURSES OF RIVERS AND OF THE SO-CALLED BAER'S LAW

Read before the Prussian Academy, January 7, 1926. Published in the German periodical, Die Naturwissenschaften, *Vol. 14, 1926.*

It is common knowledge that streams tend to curve in serpentine shapes instead of following the line of the maximum declivity of the ground. It is also well known to geographers that the rivers of the northern hemisphere tend to erode chiefly on the right side. The rivers of the southern hemisphere behave in the opposite manner (Baer's law). Many attempts have been made to explain this phenomenon, and I am not sure whether anything I say in the following pages will be new to the expert; some of my considerations are certainly known. Nevertheless, having found nobody who was thoroughly familiar with the causal relations involved, I think it is appropriate to give a short qualitative exposition of them.

First of all, it is clear that the erosion must be stronger the greater the velocity of the current where it touches the bank in question, or rather the more steeply it falls to zero at any particular point of the confining wall. This is equally true under all circumstances, whether the erosion depends on mechanical or on physico-chemical factors (decomposition of the ground). We must then concentrate our attention on the circumstances which affect the steepness of the velocity gradient at the wall.

In both cases the asymmetry as regards the fall in velocity in question is indirectly due to the formation of a circular motion to which we will next direct our attention.

I begin with a little experiment which anybody can easily repeat. Imagine a flat-bottomed cup full of tea. At the bottom there are some tea leaves, which stay there because they are rather heavier than the liquid they have displaced. If the liquid is made to rotate by a spoon, the leaves will soon collect in the center of the bottom of the cup. The explanation of this phenomenon is as follows: the rotation of the liquid causes a centrifugal force to act on it. This in itself would give rise to no change in the flow of the liquid if the latter rotated like a solid body. But in the neighborhood of the walls of the cup the liquid is restrained by friction, so that the angular velocity with which it rotates is less there than in other places nearer the center. In particular, the angular velocity of rotation, and therefore the centrifugal force, will be

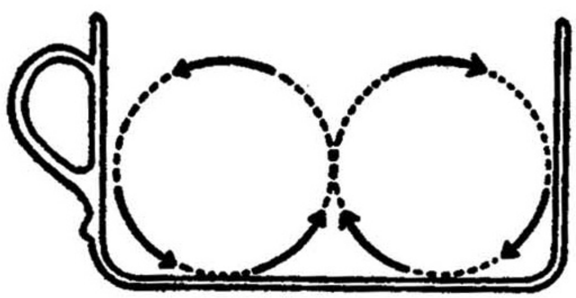

FIG. 1

smaller near the bottom than higher up. The result of this will be a circular movement of the liquid of the type illustrated in Fig. 1 which goes on increasing until, under the influence of ground friction, it becomes stationary. The tea leaves are swept into the center by the circular movement and act as proof of its existence.

The same sort of thing happens with a curving stream (Fig. 2). At every cross-section of its course, where it is bent, a centrifugal force operates in the direction of the outside of the curve (from *A* to *B*). This force is less near the bottom, where the speed of the current is reduced by friction, than higher above the bottom. This causes a circular movement of the kind illustrated in the diagram. Even where there is no

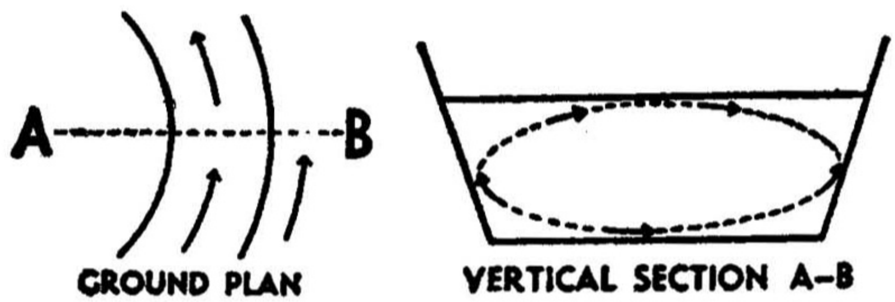

FIG. 2

bend in the river, a circular movement of the kind shown in Fig. 2 will still take place, if only on a small scale, as a result of the earth's rotation. The latter produces a Coriolis-force, acting transversely to the direction of the current, whose right-hand horizontal component amounts to $2\, v\, \Omega \sin \phi$ per unit of mass of the liquid, where v is the velocity of the current, Ω the speed of the earth's rotation, and ϕ the geographical latitude. As ground friction causes a diminution of this force toward the bottom, this force also gives rise to a circular movement of the type indicated in Fig. 2.

After this preliminary discussion we come back to the question of the distribution of velocities over the cross-section of the stream, which is the controlling factor in erosion. For this purpose we must first realize how the (turbulent) distribution of velocities develops and is maintained. If the water which was previously at rest were suddenly set in motion by the action of a uniformly distributed accelerating force, the distribution of velocities over the cross-section would at first be uniform. A distribution of velocities gradually increasing from the confining walls toward the center of the cross-section would only establish itself after a time, under the influence of friction at the walls. A disturbance of the (roughly speaking) stationary distribution of velocities over the cross-section would only gradually set in again under the influence of fluid friction.

Hydrodynamics pictures the process by which this stationary distribution of velocities is established in the following way. In a plane (potential) flow all the vortex-filaments are concentrated at the walls. They detach themselves and slowly move toward the center of the cross-section of the stream, distributing themselves over a layer of increasing thickness. The velocity gradient at the walls thereby gradually diminishes. Under the action of the internal friction of the liquid the vortex filaments in the interior of the cross-section are gradually absorbed, their place being taken by new ones which form at the wall. A quasi-stationary distribution of velocities is thus produced. The important thing for us is that the attainment of the stationary distribution of velocities is a slow process. That is why relatively insignificant, constantly operative causes are able to exert a considerable influence on the distribution of velocities over the cross-section. Let us now consider what sort of influence the circular motion due to a bend in the river or the Coriolis-force, as illustrated in Fig. 2, is bound to exert on the distribution of velocities over the cross-section of the river. The particles of liquid in most rapid motion will be farthest away from the walls, that is to say, in the upper part above the center of the bottom. These most rapid parts of the water will be driven by the circulation toward the right-hand wall, while the left-hand wall gets the water which comes from the region near the bottom and has a specially low velocity.

Hence in the case depicted in Fig. 2 the erosion is necessarily stronger on the right side than on the left. It should be noted that this explanation is essentially based on the fact that the slow circulating movement of the water exerts a considerable influence on the distribution of velocities, because the adjustment of velocities by internal friction which counteracts this consequence of the circulating movement is also a slow process.

We have now revealed the causes of the formation of meanders. Certain details can, however, also be deduced without difficulty from these facts. Erosion will be comparatively extensive not merely on the right-hand wall but also on the right half of the bottom, so that there will be a tendency to assume a profile as illustrated in Fig. 3.

Moreover, the water at the surface will come from the left-hand wall, and will therefore, on the left-hand side especially, be moving less rapidly than the water rather lower

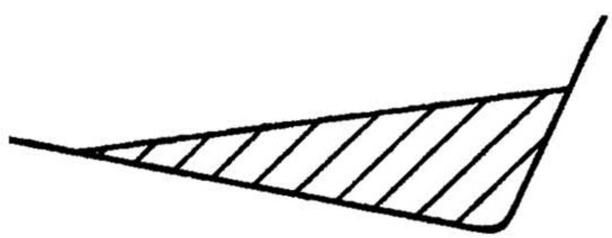

FIG. 3

down. This has, in fact, been observed. It should further be noted that the circular motion possesses inertia. The circulation will therefore only achieve its maximum beyond the place of the greatest curvature, and the same naturally applies to the asymmetry of the erosion. Hence in the course of the erosion an advance of the wave-line of the meander-formation is bound to take place in the direction of the current. Finally, the larger the cross-section of the river, the more slowly will the circular movement be absorbed by friction; the wave-line of the meander-formation will therefore increase with the cross-section of the river.