physics@mit

MEMORANDUM

To: Yorem Soreq

From: Peter Fisher

Subject: Nuclear selection rules in $(N, Z)^* \to (N, Z) + e^+ + e^-$

Date: January 29, 2017

Equation references are to Tomoda, 1991.

From Eq. 3.44, $\alpha \neq \beta$ for the allowed process $(N, Z)^* \to (N, Z) + e^+ + e^-$. This is equivalent to introducing a right handed current for a Majorana neutrino with $\lambda \neq 0$, $\kappa = \eta = 0$ in Eq. 3.40.

Following along, in Eq. 3.50, $X_1 = X_5 = X_6 = 0$. The resulting nuclear currents are given in Table 4, reproduced below. They are of the for 1 and $\sigma_1 \cdot \sigma_2$. The σ's operate on the quark spins, so the nuclear currents allow ΔJ=0,1, or 2 in the nuclear angular momentum. This seems to preclude $^{178m2}\text{Hf} \to ^{178}\text{Hf} + e^+ + e^-$ unless there is some mixing between angular momentum states.

Table 4. Factor decomposition of the $0^+ \to 0^+$ $0\nu\beta\beta$ decay amplitudes into various contributions. Only the relative magnitudes are given for each column. Here ω and $\boldsymbol{k}$ are the energy and the momentum of the virtual light neutrinos, $k = |\boldsymbol{k}| \approx \omega$, and ϵ_i the energies of the emitted electrons. The unit vectors $\hat{\boldsymbol{r}}_{12}$ and $\hat{\boldsymbol{r}}_{+12}$, which originate from the momentum $\boldsymbol{k}$ of the neutrino ($\hat{\boldsymbol{r}}_{12}$) and the electron P-wave ($\hat{\boldsymbol{r}}_{12}$ and $\hat{\boldsymbol{r}}_{+12}$), are included in the contributions of the nuclear currents in order to show their way of coupling to $\boldsymbol{\sigma}$ and $\boldsymbol{D}$. The common factor $\tau_1^+ \tau_2^+$ has been omitted in the last column. See text for the definition of S_i and P_i.

Type of matrix element	(a) Neutrino propagator	(b) Electron wavefunctions	(c) Nuclear currents
X_1 $(M_{GT}^{(0\nu)}, \chi_F)$	$\langle m_\nu \rangle / \omega^2$	$\sqrt{2} S_1 S_2$	$g_V^2 - g_A^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
X_1 (χ_{GTh}, χ_{Fh})	$\langle\langle m_\nu^{-1} \rangle\rangle$		
X_3 $(\tilde{\chi}_{GT}, \tilde{\chi}_F)$	$\langle \lambda \rangle (\epsilon_1 - \epsilon_2)/2\omega^2$	$2 S_1 S_2$	$g_V^2 \mp g_A^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
	$\langle \eta \rangle (\epsilon_1 - \epsilon_2)/2\omega^2$		
X_4 $(\chi'_{GT}, \chi'_F, \chi'_T)$	$\langle \lambda \rangle k/\omega^2$	$(S_1 P_2 - P_1 S_2) r_{12}/R$	$g_V^2 \pm g_A^2 (\frac{1}{3} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - 2 S_{12})$
	$\langle \eta \rangle k/\omega^2$		
X_5 (χ'_P)	$\langle \eta \rangle k/\omega^2$	$(S_1 P_2 + P_1 S_2) r_{+12}/R$	$g_V g_A (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot (\hat{\boldsymbol{r}}_{12} \times \hat{\boldsymbol{r}}_{+12})$
X_6 (χ'_R)	$\langle \eta \rangle k/\omega^2$	$2 S_1 S_2$	$g_V g_A \hat{\boldsymbol{r}}_{12} \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{D}_2 - \boldsymbol{\sigma}_2 \times \boldsymbol{D}_1)$