physics@mit

MEMORANDUM

To: Mike Spiser

From: Peter Fisher

Subject: Models falsified by population measurements in yeast

Date: February 27, 2017

The question arose during Science Council on Feb. 16 as to what models had been falsified by Jeff Gore's measurements of yet populations. The work carried out over six papers validated the prevailing view that fold bifurcations described the population dynamics of many living systems, but there were completing ideas that were not supported.

Malthus in 1798 put forward the ideas that led to logistic population growth,

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right),$$

where r defines the growth rate and K is the carrying capacity for the population P. With time, the population stabilizes at $P = K$. As the population increases, competition for resources increased and the growth of the populations slowed increased.

In the 1950's, Allee put forward the notation that a population's growth rate increases with population density up to a certain population density, after which the growth rate decreases. Fig. 1 compares logistic and Allee replacement rates.

The gold curve in Fig. 1 shows the replacement rate for a given set of environmental parameters. The curve changes as, for example, the temperature changes. If the replacement rate changes so there are no zero crossings, Fig. 2, a fold bifurcation results.

Fig. 3 shows a different possibility in which changes in the environments parameter moves the left-most zero crossing below $P = 0$, resulting in the loss of the unstable fixed point. The system has stable fixed points at higher values of t.

Gore's work showed the yeast and other systems he experimented with followed fold type bifurcations, leading to the population collapse owing to the loss of all fixed points rather than the type of collapse illustrated by Fig. 3 in which the unstable fixed point is lost and the stable fixed point tends to zero.

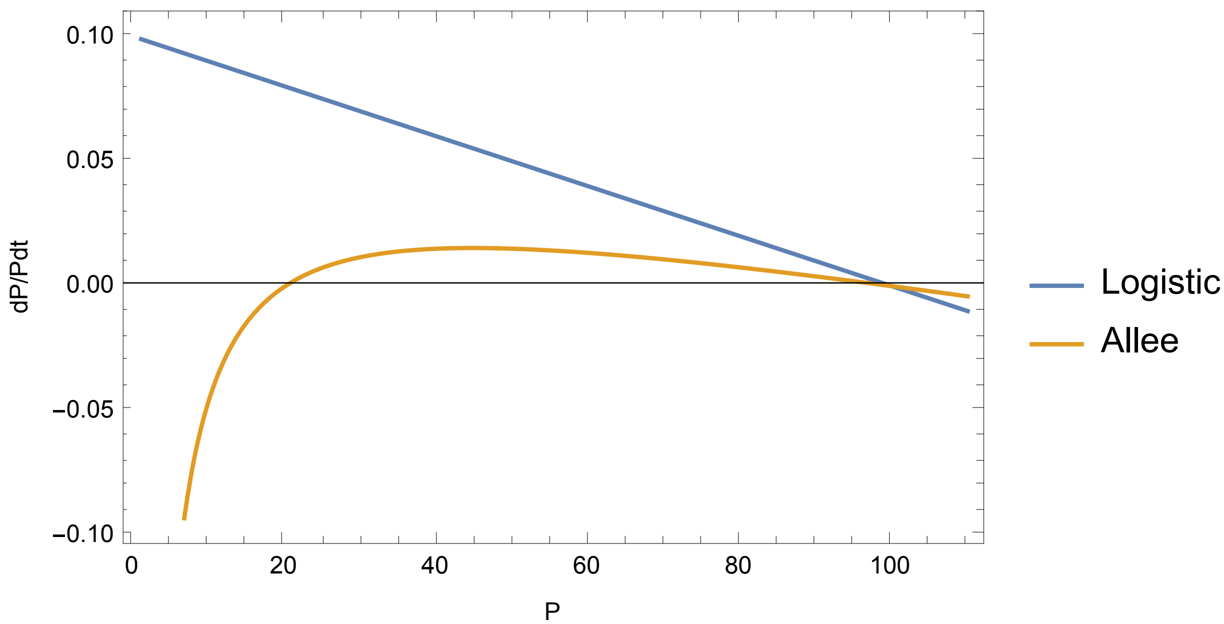

Figure 1: Comparision with logistic (blue) and Allee (gold) replacement rates, $r = (1/P)\,dP/dt$. For populations below 100, in logistic growth, the population increases at an ever diminishing rate until $P = 100$. In Allee growth, if $P < 20$, $r < 0$ and the population dies away. Otherwise, the population stabilizes at $P = 100$.

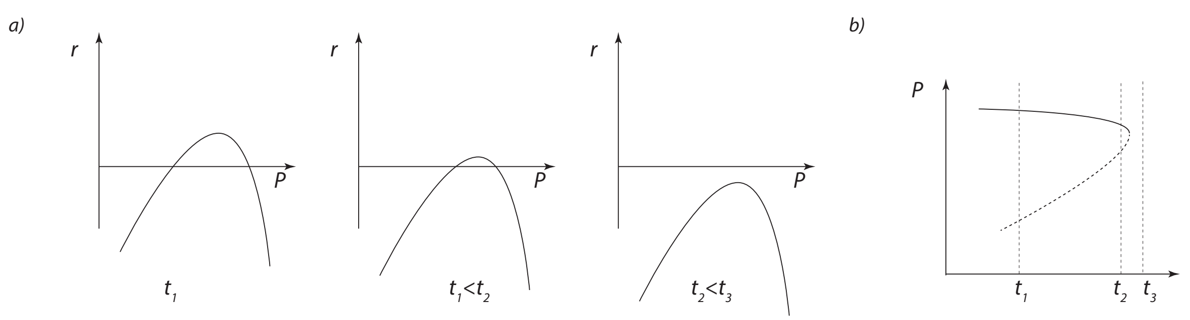

Figure 2: a) Three successive cures with increasing environmental parameter t. for the first two curves, the left-most intersection of the curve with the x axis is an unstable fixed point, which the right most is a stable fixed point. The rightmost curve has no fixed points: all populations eventually die away. b) The solid line indicates stable populations as a function of parameter t while the dotted line indicates unstable populations. Past a certain value of t, there are no fixed points.

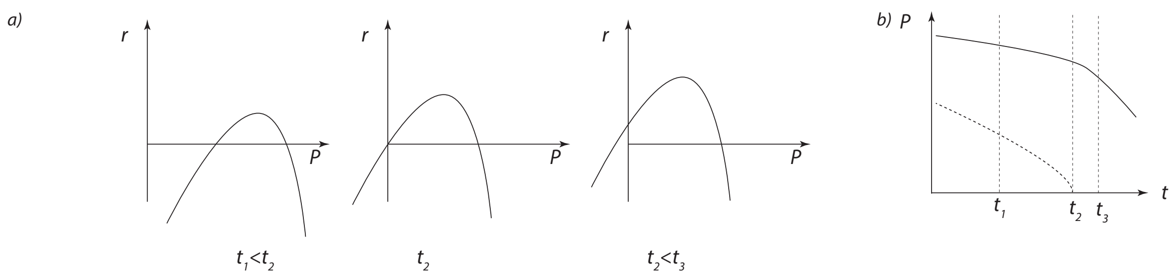

Figure 3: a) Shows three successive replacement curves at a function of t. At t_2, the left-most unstable fixed point reaches $P = 0$, meaning there are no unstable fixed pints for $t > t_2$. b) The solid line indicates the stable fixed point as a function of parameter t, the dashed indicates the unstable fixed points that disappears for $t > t_2$.