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MEMORANDUM

To: Curt Marble, Ross Corliss

From: Peter Fisher

Subject: Motion of Moon's shadow on Earth during the 2017 total solar eclipse

Date: October 15, 2017

During the 2017 total solar eclipse, the Moon's shadow moved west-to-east across the US, Fig 1, at odds with our intuition, that tells use the Earth's rotation would cause the shadow to move east to west.

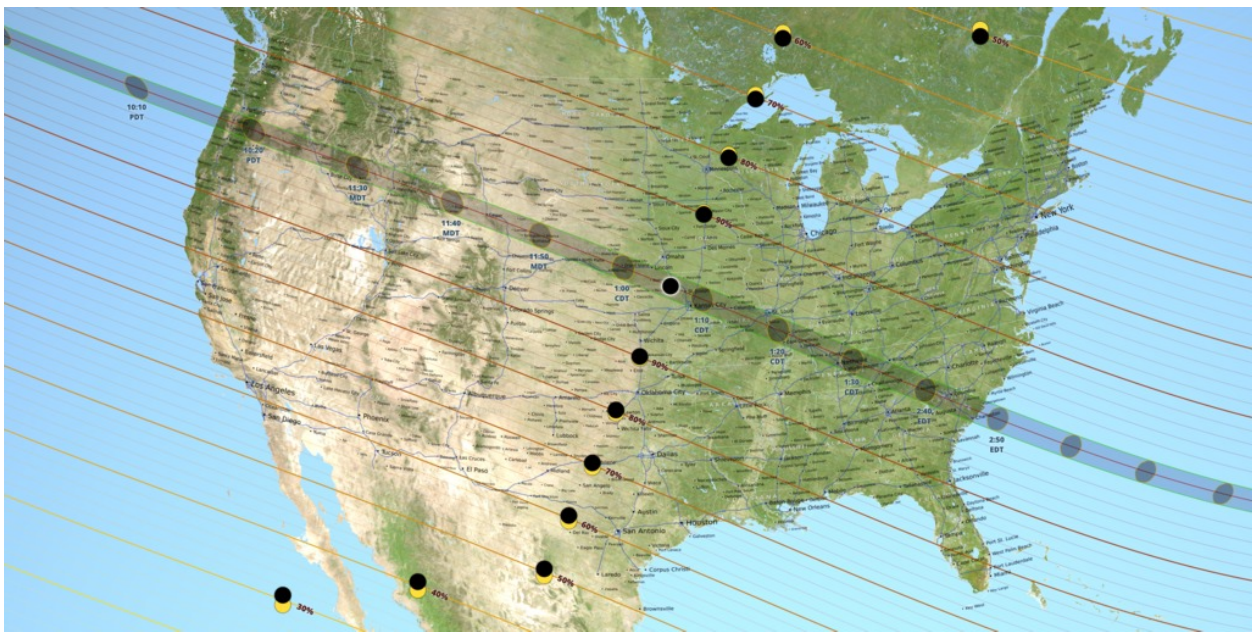

Figure 1: Map showing path of the 2017 total eclipse. Source:NASA

Assuming all orbits are circular (reasonable in this case), the Earth orbits the Sun at a speed of $v_e = 2 \times \pi \times 150,000,000/\left(365.25 \times 23.93 \times 60 \times 60\right) = 29.9$ km/s, the Moon orbits the Earth at $v_m = 2 \times \pi \times 385,000/\left(27.321 \times 23.93 \times 60 \times 60\right) =$1.02 km/s, and a point on the equator moves at $v_r = 2 \times \pi \times 6371/\left(23.93 \times 60 \times 60\right) =$0.46 km/s[1], Fig. 2A.

At the time of the eclipse (referred to as syzygy), the Sun, Moon, and Earth lie along one line with the Moon between the Earth and Sun, casting its shadow on Earth, Fig. 2B. Choose a frame centered on the Sun in which the Earth orbits with speed v_e. The a point on the equator (crossing

[1]The refers to a point on the equator as it passes through the plane of the Sun-Moon orbit.

the Earth-Sun orbit plane), moves with velocity $v_r' = v_e - v_r$ and the Moon moves with velocity $v_m' = v_m - v_e$. Next, choose an inertial frame instantaneously at rest with the point on the equator, Fig. 2 C. This frame moves with speed $u = v_r' = v_e - v_r$ relative to the Sun centered frame. In this frame, the Moon moves with speed $v_m' - u = v_m - v_e - v_r + v_e = v_m - v_r = 1.02\,\mathrm{km/s} - 0.46\,\mathrm{km/s} = 0.56\,\mathrm{km/s}$ in the west-to-east direction, Fig. 3. The Sun's motion also causes the shadow to move at a speed of $(v_e + v_m - 2v_r)\, r_{em}/r_{ms} = 0.078$ km/s, Fig. 3, which gives a total of 0.64 km/s.

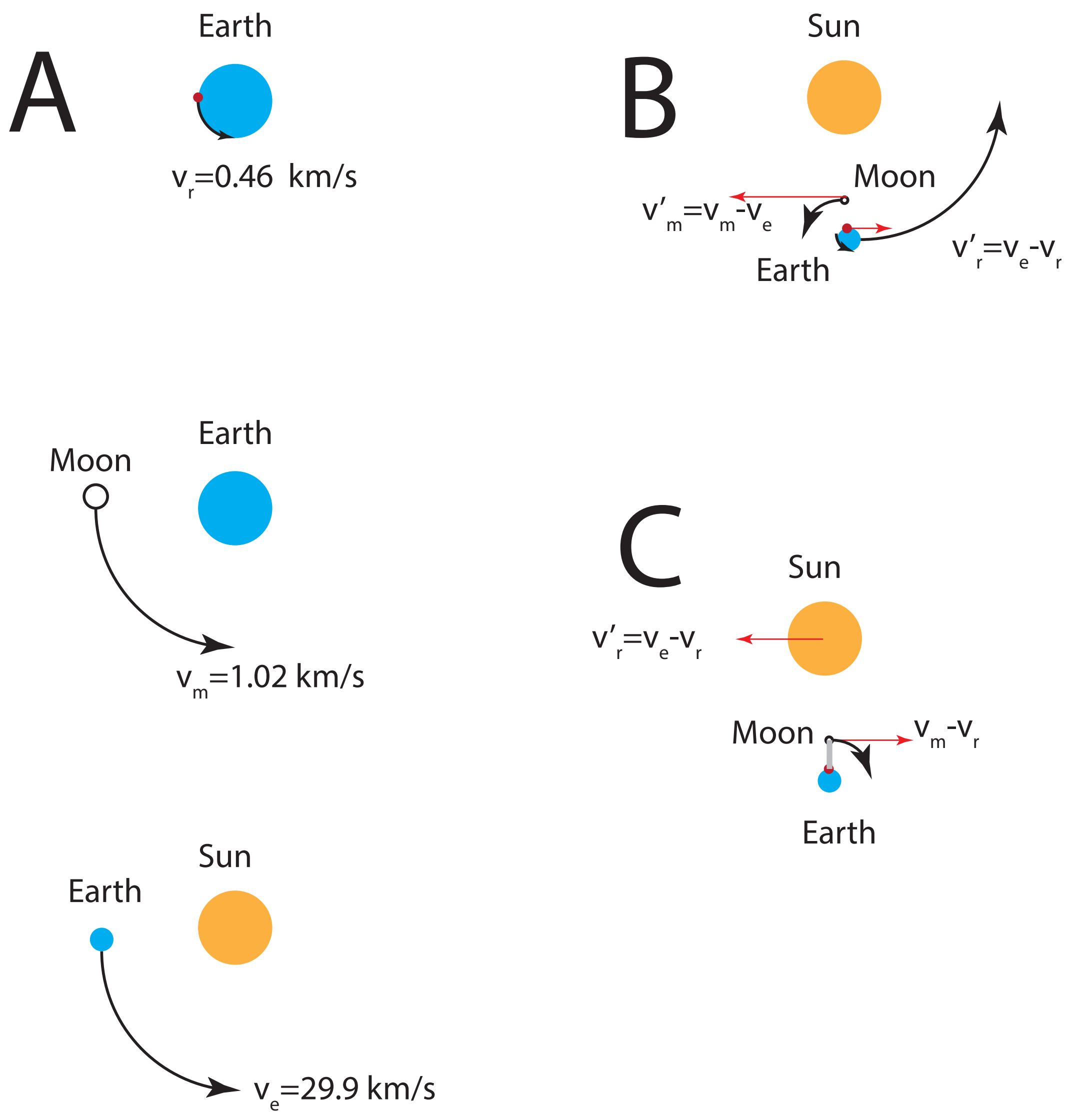

Figure 2: A) Velocities used in the problem, each measured relative to the rest frame of largest body relative to the fixed stars. B) Velocities at syzygy relative to the rest frame of the Sun. C) Velocities at syzygy taking the point on Earth nearest the Moon at rest.

The speed of sound, Mach 1, is 0.3 km/s, so the shadow moves at a minimum speed of Mach 2.1 across the Earth, a speed accessible to only the fastest airplanes. The angle of the Earth's axis at the time of the eclipse and the latitude of the observer can both to make v_r smaller. The Earth's rotation axis tilts 23° with respect to the plane of the Earth's orbit and the shadow of the eclipse was in the middle Northern latitudes, so a better value is $v_{r,corr} = v_r \sin 45^\circ = 0.33$ km/s, which gives a shadow velocity of 0.81 km/s or Mach 2.7.

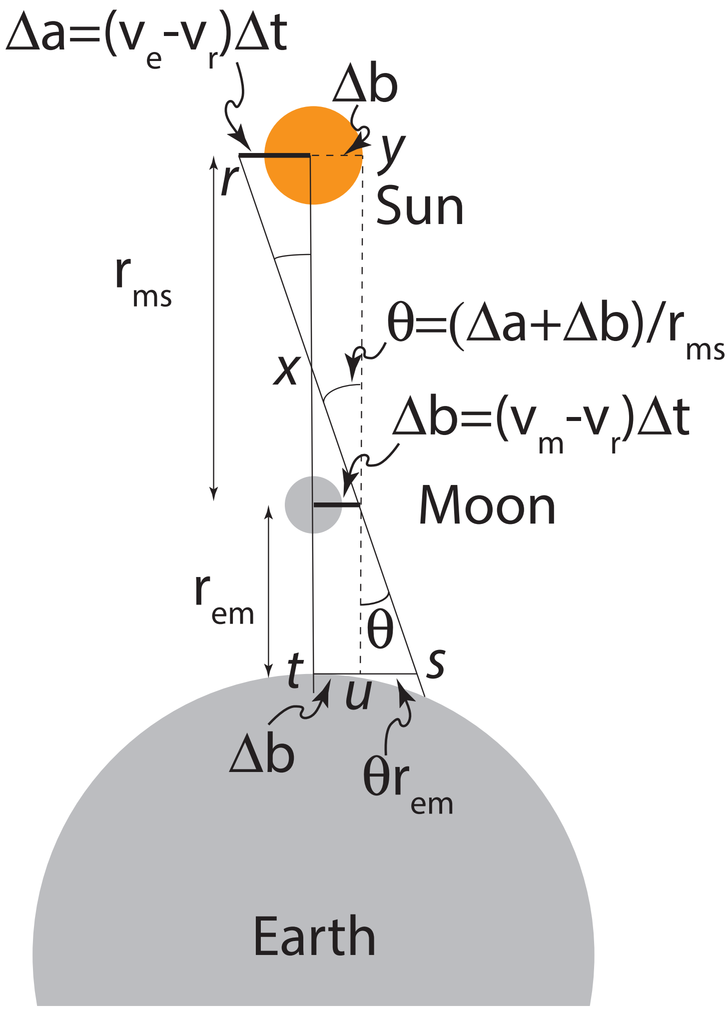

Figure 3: Geometry used to find speed of the Moon's shadow on the Earth's surface. The analysis proceeds as follows: in time dt in the frame in which the red point on Earth is at rest, the Sun moves a distance $\Delta a = (v_e - v_r)\, dt$ to the left and the Moon moves $\Delta b = (v_m - v_r)\, dt$ to the right. The diagonal line $r - s$ shows the shadow of the Moon on the Earth after dt. The line $t - -s$ has two segments: $t - u$, which has length Δb and results from the Moon's motion, and $u - s$ which has length θr_{em} and results from the Sun's motion. The angle θ is found from the triangle $r - -x - -y$ which has opposite side of length $\Delta a + \Delta b$ and adjacent side of length r_{ms}. Then, $\tan\theta \sim \theta = (\Delta a + \Delta b)\, / r_{ms}$. Putting it all together, the distance the shadow moves in time Δt is $\Delta b + (\Delta a + \Delta b)\, r_{em}/r_{ms}$, which gives a velocity of $(v_m - v_r) + (v_e + v_m - 2v_r)\, r_{em}/r_{ms}$