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MEMORANDUM

To: Mark Longtin

From: Peter Fisher

Subject: Procedure for computing the cosine of an angle in a Curta

Date: Sunday, May 8, 2016

Using the method developed in the previous memo, this memo describes the procedure for computing the cosine of an angle expressed in radians with two digits of precision.

In this example, I show how to compute $\cos 68°$. Table 1 shows the sequence of operations to find $\cos 68°$ using the summation method from the companion memo. There are a total of 29 Curta operations. In addition, the user must memorize the starting numbers and perform a complex calculation to find them. It is *possible* to carry out this calculation without writing anything down.

For comparison, the direct calculation of $\cos x \sim 1 - x^{/}2 + x^4/24$ proceeds as follows:

1. Convert to radians: $68 \times \pi = 21352$, $21352/180 = 119$, three operations
2. Compute x^2: $x^2 = 119^2 = 14161$, compute $x^4 = 14161^2 = 200533921$, two operations
3. Compute $x^4/24$: $200533921/24 = 8355580$, one operation
4. Compute $x^2/2$: $14161/2 = 7080$, one operation (if you don't do it in your head)
5. Compute $1 - x^2/2 + x^4/24$: $100000000 - 708000000 + 8355580 = 37555580$, two operations. Which gives $\cos 68° = 0.37555$. The true value is 0.37461 or 0.2% difference.

The direct calculation of the second order Taylor expansion gives a better result in about a third the number of operations.

Step	Operation	Example	
1	Convert angle to radians	$x \to x\pi/180$	1.19
2	Multiply by 100, truncate	$k = \lfloor(100x)\rfloor$	119
3	Square	k^2	14,161
4	Extract digits	$x_5 = \lfloor(k^2/10^5)\rfloor$	$x_5 = 0$
		$x_i = \lfloor(k^2 - \sum_{j=i+1}^{5} x_j 10^j)/10^i\rfloor$	$x_4 = 1$
			$x_3 = 4$
			$x_2 = 1$
			$x_1 = 6$
			$x_0 = 1$
5	Enter constant term	12 000 000 000	12 000 000 000
6	First digit x_5	$-10\,000\,000\,000$	12 000 000 000
	Add $x_5 = 0$ times increment by	90 000 000 000	
7	Second digit x_4	$-5\,500\,000\,000$	6 500 000 000
		$+10\,000\,000\,000x_5$	
	Add $x_4 = 1$ times increment by	1 000 000 000	
8	Third digit x_3	$-595\,000\,000$	5 045 000 000
		$+1\,000\,000\,000x_5$	
		$+100\,000\,000x_4$	
	Add $x_3 = 4$ times increment by	10 000 000	
8	Fourth digit x_2	$-59\,950\,000$	4 626 050 000
		$+100\,000\,000x_5$	
		$+10\,000\,000x_4$	
		$+1\,000\,000x_3$	
	Add $x_2 = 1$ times increment by	100 000	
8	Third digit x_1	$-5\,999\,500$	4 616 832 000
		$+10\,000\,000x_5$	
		$+1\,000\,000x_4$	
		$+100\,000x_3$	
		$+10\,000x_2$	
	Add $x_1 = 6$ times increment by	1 000	
9	Sixth digit x_0	$-599\,995$	4 614 988 880
		$+1\,000\,000x_5$	
		$+100\,000x_4$	
		$+10\,000x_3$	
		$+1\,000x_2$	
		$+100x_1$	
	Add $x_0 = 1$ times increment by	10	
10	Divide by constant term	12 000 000 000	0.37

Table 1: Sequence of operations using the summation method.

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Step	r	Sum	Calc	Var.
1	−5 500 000 000	6 500 000 000	0.5417	−0.279
2	−5 995 000 000	6 005 000 000	0.5004	−0.219
3	−6 480 000 000	5 520 000 000	0.4600	−0.151
4	−6 955 000 000	5 045 000 000	0.4204	−0.071
5	−7 001 950 000	4 998 050 000	0.4165	−0.062
6	−7 048 800 000	4 951 200 000	0.4126	−0.053
7	−7 095 550 000	4 904 450 000	0.4087	−0.044
8	−7 142 200 000	4 857 800 000	0.4048	−0.035
9	−7 188 750 000	4 811 250 000	0.4009	−0.025
10	−7 235 200 000	4 764 800 000	0.3971	−0.016
11	−7 281 550 000	4 718 450 000	0.3932	−0.006
12	−7 327 800 000	4 672 200 000	0.3894	0.004
13	−7 373 950 000	4 626 050 000	0.3855	0.014
14	−7 378 559 500	4 621 440 500	0.3851	0.015
15	−7 383 168 000	4 616 832 000	0.3847	0.016
16	−7 383 628 795	4 616 371 205	0.3847	0.016
17	−7 384 089 580	4 615 910 420	0.3847	0.016
18	−7 384 550 355	4 615 449 645	0.3846	0.016
19	−7 385 011 120	4 614 988 880	0.3846	0.016

Figure 1: Table of operations. r is the value subtracted at each step, Sum is the current value of the evaluation, $Calc$ is the current calculated value of $\cos 68°$ and Var is the fractional variance from the true value.

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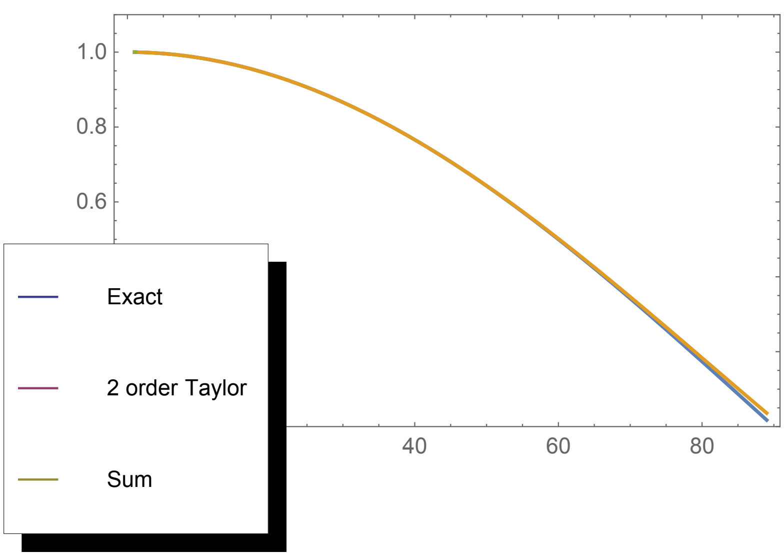

Figure 2: Comparison of levels of calculation.