MEMORANDUM

To: Mark Longtin

From: Peter Fisher

Subject: Procedure for computing quadratic equations ont he Curta

Date: Friday, May 13, 2016

The Curta is designed for addition of numbers. Subtraction is easy, multiplication becomes difficult and division is quit difficult from an operation point of view. An interesting question is, "How can you use the Curta to compute more complicated quantities, like $\sqrt{x}, \cos x, \ln x$ and so on?" After suitable scaling so x is small, we can evaluate $f(x) = u + tx + sx^2$

Two digits of precision are sufficient for most applications and we can relate a quadratic expression to an incrementing sum

$$c + \sum_{i=1}^{n} ai + b = (an + b) + ... + (a + b) = \frac{an^2}{2} + \frac{an}{2} + bn = n^2\left(\frac{a}{2}\right) + n\left(\frac{a}{2} + b\right) + c \quad (1)$$

$$= sn^2 + tn + u \quad (2)$$

which leads to

$$a = 2s \quad (3)$$

$$b = t - s \quad (4)$$

$$c = c \quad (5)$$

We have to shift the decimal point since the Curta operates on integers. For two digit precision, $k = \lfloor 100x \rfloor$ and $10\,000f(x) = 10\,000u + 100tk + sk^2$ and

$$a = 2s \quad (6)$$

$$b = 100t - s \quad (7)$$

$$c = 10\,000u \quad (8)$$

If $f(x) = u + tx^2 + sx^4$, then $100\,000\,000f(x) = 100\,000\,000u + 10\,000tk^2 + sk^4$.

What we really want to do is find the *digits* $x_0...x_m$ of $k = x_0 + 10x_1 + ... + 10^m x_m$. To be concrete, we take $m = 5$, six digit numbers. Then,

$$\sum_{i=1}^{k} ai + b = \sum_{1}^{x_5 10^5} ai + b + \sum_{x_5 10^5+1}^{x_5 10^5+x_4 10^4} ai + b + \sum_{x_5 10^5+x_4 10^4+1}^{x_5 10^5+x_4 10^4+x_3 10^3} ai + b \quad (9)$$

$$+ \sum_{x_5 10^5+x_4 10^4+x_3 10^3+1}^{x_5 10^5+x_4 10^4+x_3 10^3+x_2 10^2} ai + b + ... \quad (10)$$

$$= r_5 + r_4 + r + 3 + r + 2 + r_1 + r_0 \quad (11)$$

The individual sums may be computed in the usual way in a straightforward, but tedious process. The result is

$$
\begin{aligned}
r_5 &= 5\,000\,050\,000a + 100\,000b \\
&\quad \text{in steps of } 10\,000\,000\,000a\ x_5 - 1 \text{ times} \\
r_4 &= 50\,005\,000a + 10\,000b \\
&\quad \text{in steps of } 100\,000\,000a\ x_4 - 1 \text{ times} \\
r_3 &= 500\,500a + 1\,000b + 10\,000\,000ax_4 + 100\,000\,000ax_5 \\
&\quad \text{in steps of } 1\,000\,000a\ x_3 - 1 \text{ times} \\
r_2 &= 5\,050a + 100b + 100\,000ax_3 + 1\,000\,000ax_4 + 10\,000\,000ax_5 \\
&\quad \text{in steps of } 10\,000a\ x_2 - 1 \text{ times} \\
r_1 &= 55a + 10b + 10\,00ax_2 + 1\,000ax_3 + 100\,000ax_4 + 1\,000\,000ax_5 \\
&\quad \text{in steps of } 100a\ x_1 - 1 \text{ times} \\
r_0 &= a + b + 10ax_1 + 100ax_2 + 1\,000ax_3 + 10\,000ax_4 + 100\,000ax_5 \\
&\quad \text{in steps of } a\ x_0 - 1 \text{ times}
\end{aligned}
$$