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MEMORANDUM

To: ALCON

From: Peter Fisher

Subject: Compton Scattering Kinematics

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Compton scattering is the process $\gamma + e \to \gamma + e$ and was an early indication of the quantization of light. Fig. 1 shows the kinematics. The four vectors of the initial and final states are,

$$
\begin{aligned}
k &= (E, 0, 0, E) \\
k' &= \left(E', E' \sin\theta, 0, E' \cos\theta\right) \\
p &= (m, 0, 0, m) \\
p' &= \left(\mathcal{E}', \mathcal{E}' \sin\phi, 0, \mathcal{E}' \cos\phi\right).
\end{aligned}
$$

Momentum and energy conservation require,

$$
\begin{aligned}
k + p &= k' + p' \\
E + m &= E' + \mathcal{E}'.
\end{aligned}
$$

Then,

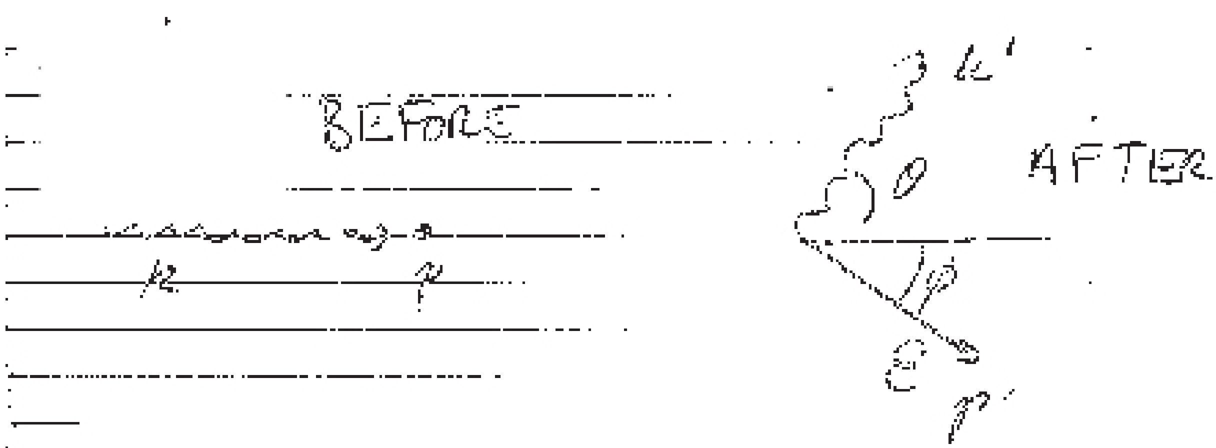

Figure 1: Kinematics and variables for Compton scattering.

$$
\begin{aligned}
k - k' &= p' - p \\
\left(k - k'\right)^2 &= \left(p' - p\right)^2 \\
k^2 - 2kk' + k'^2 &= p^2 - 2pp'^2 + p'^2 \\
0 + kk' + 0 &= -m^2 + pp' \\
EE' \cos\theta &= m\mathcal{E}'
\end{aligned}
$$

Energy conservation says, $E + m = E' + \mathcal{E}'$ and,

$$
\begin{aligned}
EE' \cos\theta &= m\left(E + m - E'\right) \\
E' &= \frac{mE}{E\left(1 - \cos\theta\right) + m}
\end{aligned}
$$

gives the energy of the scattered photon. Usually, the kinetic energy of the recoiling electron is detected, $\mathcal{K}' = \mathcal{E}' - m = E - E'$,

$$
\mathcal{K}' = \frac{E^2\left(1 - \cos\theta\right)}{E\left(1 - cos\theta\right) + m}.
$$

From inspection of Eq. 1, the energy of the recoil photon is lowest with $\theta = \pi$, giving the largest energy transfer to the electron,

$$
\begin{aligned}
\mathcal{K}'_{max} &= \frac{2E^2}{2E + m} = \frac{E}{1 + m/2E} \\
&\rightarrow E - \frac{m}{2} \text{ if } E >> m,
\end{aligned}
$$

Fig. 2.

Figure 2: Pulse height spectrum for electron recoils in a detector. Compton edge is shown which lies $m/2$ below the full energy peak when $E >> m$.