physics@mit

# MEMORANDUM

**To:** ALCON

**From:** Peter Fisher

**Subject:** Electromagnetic Units and Relations for a Charged Particle Orbiting in a Magnetic Field

**Date:** October 31, 2018

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A particle with an initial velocity in the $x-y$ plane will orbit in that plane if there is a magnetic field along the $z$ axis. The note uses SI units and all motion takes place in the $x-y$ plane.. The famous formula relating the momentum and orbit parameters is,

$$p = \frac{0.3\mathrm{GeV/c}}{\mathrm{T-m}} BR, \tag{1}$$

where $B$ is the magnetic field strength and $R$ is the orbit radius. The prefactor is just the electron charge in odd units found in the following way: from the Lorentz force law,

$$\vec{F} = e\left(\vec{v}\times\vec{B}\right) + \vec{E}).$$

If $\hat{v} = \hat{E}\times\hat{B}$ then $v = E/B$. If $E = 1V/m$ and $B = 1T$, then $v$ =1 m/s= 1 V/T-m. Then $1V/T = 1\mathrm{m}^2/\mathrm{s}$. Eq. 1 is,

$$\frac{0.3\mathrm{GeV/c}}{\mathrm{T-m}} = \frac{3\times 10^8}{\mathrm{mc}} e\frac{V}{T} = e\frac{3\times 10^8\mathrm{m}^2/\mathrm{s}}{\mathrm{mc}} = e.$$

The orbit frequency comes from using $p = m\gamma\beta c$ and solving for $\beta$,

$$\beta = \frac{pc}{E} = \frac{pc}{\sqrt{m^2c^2 + p^2c^2}} = \frac{eRB}{\sqrt{(m^2c^2 + (eRB)^2}}.$$

The orbit frequency is $\omega = \beta c/R$ and the acceleration $a = R\omega^2$ or,

$$a = \frac{c^2}{R}\frac{(eBR)^2}{m^2c^2 + (eBR)^2} \tag{2}$$

$$= \frac{c^2}{R} \text{ if } eBR >> mc \tag{3}$$

$$= \frac{e^2B^2R}{m^2} \text{ if } eBR << mc \tag{4}$$

For a given particle species with charge $q$ and magnetic field, the value of $R$ that gives the maximum acceleration is,

$$R_{max} = \pm\frac{mc}{qB}$$

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and the maximum acceleration is,

$$a_{max} = \frac{Bcq}{2m}.$$

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