MEMORANDUM

To: ALCON

From: Peter

Subject: Electromagnetic Shielding

Date: November 18, 2020

---

1 Maxwell's Macroscopic Equations

$$
\begin{aligned}
&\vec{\nabla} \cdot \vec{D} = \rho \\
&\vec{\nabla} \times \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial \vec{t}} = \vec{J} \\
&\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \\
&\vec{\nabla} \cdot \vec{B} = 0 \\
&\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}
\end{aligned}
$$

The charge density ρ and current density $\vec{J}$ refer to macroscopically free charges and currents.

2 Quantities and dimensions

Si base units

1. Second (s)
2. Meter (m)
3. Kilogram (kg)
4. Ampere (A)
5. Kelvin (K)
6. Mole (mol)
7. Candela (cd)

Electromagnetic quantities and units

1. $\vec{D} = \epsilon_\circ \vec{E} + \vec{P}$ - A-s/m^2 - electric displacement
2. V - J/A-s=V - potential

3. $\vec{H}$ - A/m - magnetic field $\vec{H} = \vec{B}/\mu_\circ - \vec{M}$
4. $\vec{B}$ - N/A-m-s=T - magnetic induction
5. $\vec{E}$ - N/A-s=V/m - electric field
6. R - Ω=V/A=J/A^2-s - resistance
7. L - V-s/A=Ω-s=H - inductance
8. $\mu_\circ = 4\pi \times 10^{-7}$ T-m/A, magnetic permeability
9. $\epsilon_\circ = 10^7/4\pi c^2$A^2-s^2/N-m=(A-s/V-m), electric permittivity
10. $\vec{P} = \chi_e \vec{E}$ - A-s/m^2 - electric polarization
11. χ_e - A^2-s^2/N-m=(A-s/V-m) - electric susceptibility
12. $\sigma = 1/\rho$ - S/m=1/Ω-m(N/A^2-s) - conductivity
13. f_i - m - oscillator strength

3 Skin Depth

EM waves have solutions of the form

$$u(\vec{x}, t) \propto \exp\left(i\vec{k} \cdot \vec{x} - i\omega t\right)$$

where u may be the electric or magnetic field.

$$k = \frac{\omega}{v} = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c}\sqrt{1 + \frac{\chi_e}{\epsilon_\circ}} = \beta + i\frac{\alpha}{2}.$$

where $\mu = \mu_\circ$, Then,

$$u \propto \exp\left(\hat{\vec{k}}\vec{x}\left(i\beta - \frac{\alpha}{2} - i\omega t\right)\right).$$

Take $z = \hat{\vec{k}}\vec{x}$. The fields attenuate along z as $\exp(-z\alpha/2)$ and the power flux goes as $\exp(-\alpha z)$.

The lowest order model of the electric susceptibility takes each electron in the atoms comprising bulk material as an independent, driven, damped harmonic oscillator with strength f_i,

$$\chi_e = \frac{Ne^2}{m}\left[\frac{f_\circ}{w_\circ^2 - \omega^2 + i\omega\gamma_\circ} + \sum_{i=0}^{Z} \frac{f_i}{w_i^2 - \omega^2 + i\omega\gamma_i}\right]$$

where f_i is the oscillator strength. The lowest oscillator level determines whether the material is a conductor ($\omega_\circ = 0$) or an insulator ($\omega_\circ \neq 0$). For $\omega_\circ = 0$, the spring constant in the simple harmonic oscillator equation must be zero, implying the electron is unbound. For $\omega = 0$, the case of a constant electric field, the electron reaches an terminal velocity of $eE/\gamma_\circ$ and for an electron number density of N, the current density is $J = \left(Ne^2/m\gamma_\circ\right)E$, implying the conductivity for a constant field is $\sigma(\omega = 0) = Ne^2/m\gamma_\circ$. The general expression is,

$$\sigma(\omega) = \frac{Ne^2}{m}\frac{f_\circ}{\omega - i\gamma_\circ}.$$

Then,

$$\epsilon_\circ + \chi_\circ = \epsilon_\circ + i\frac{\sigma}{\omega} + \frac{Ne^2}{m}\left[\sum_{i=0}^{Z}\frac{f_i}{w_i^2 - \omega^2 + i\omega\gamma_i}\right] = \epsilon\left(\omega\right) + i\frac{\sigma}{\omega}.$$

The electric permittivity is predominantly real except when $|\omega - \omega_i| \sim \gamma_i$, area of strong scattering by atomic (or molecular) transitions. In the following, $\epsilon\left(\omega\right)$ is taken to be always real. Then, if $\beta \gg \alpha$,

The,

$$k = \omega\sqrt{\mu_\circ}\sqrt{\epsilon + i\frac{\sigma}{\omega}} = \omega\sqrt{\mu_\circ\epsilon}\sqrt{1 + i\frac{\sigma}{\omega\epsilon}}.$$

$\sigma/\omega\epsilon \gg 1$ for a good conductor and

$$k = \left(1 + i\right)\sqrt{\frac{\omega\mu_\circ\sigma}{2}}$$

and the skin depth $\delta = 2/\alpha = \sqrt{2/\omega\mu_\circ\sigma}$ gives both the attenuation length and the wavelength through $\lambda = 2\pi/\mathrm{Re}\, k = \sqrt{8\pi^2/\omega\mu_\circ\sigma}$. For poor conductors, $\sigma/\mu_\circ\epsilon \ll 1$, $k = \omega/v + i\sigma/2\sqrt{\mu_\circ/\epsilon}$, making the attenuation frequency independent.

4 Parallel conductor model

The outer surface of each plate lies at $\pm X/2$. The magnetic field outside the plates $H_\circ \exp -i\omega t\hat{z}$ and inside the plates $\left(H + iH_q\right)\exp i\omega t\hat{z}$. To first order, the time varying magnetic field induces an electric field between the plates $\left(E + iE_q\right)\exp i\omega t\hat{y}$ that drives a current density $\left(J + iJ_q\right)\exp i\omega t\hat{y}$ on the plate at

Taking a loop 1 in the x-z plane, Fig. 1 gives

$$H - iH_q - H_o = -\left(J + iJ_q\right)l = \left(\mathcal{J} + i\mathcal{J}_{\mathrm{II}}\right) \tag{1}$$

where l is the thickness of each plate.

Place the origin midway between the plates. Take a loop 2 in the y-z plane with one leg along the y-axis in the y direction and the other a distance x from the z-axis in the -y direction. Then,

$$-\left(E\left(x\right) + iE_q\left(x\right)\right) - \mu_\circ\omega\left(H + iH_q\right)x = 0 \tag{2}$$

From Ohm's law, on the plate at $-X/2$,

$$E\left(-X/2\right) + i\left(E(-X/2\right) = \frac{J + iJ_q}{\sigma} = \frac{\mathcal{J} + i\mathcal{J}_{\mathrm{II}}}{l\sigma}. \tag{3}$$

The real and imaginary parts of Eq.1-3 give six equations and solving for the magnetic field between the plates gives with $m = Xl/\delta^2$

$$\frac{H}{H_\circ} = \frac{1}{1 + m^2} \tag{4}$$

$$\frac{H_q}{H_\circ} = \frac{m}{1 + m^2}. \tag{5}$$

For a good conductor with $m \gg 1$, $H_q \sim H_\circ/m$ and $H \sim H_\circ/m^2$.

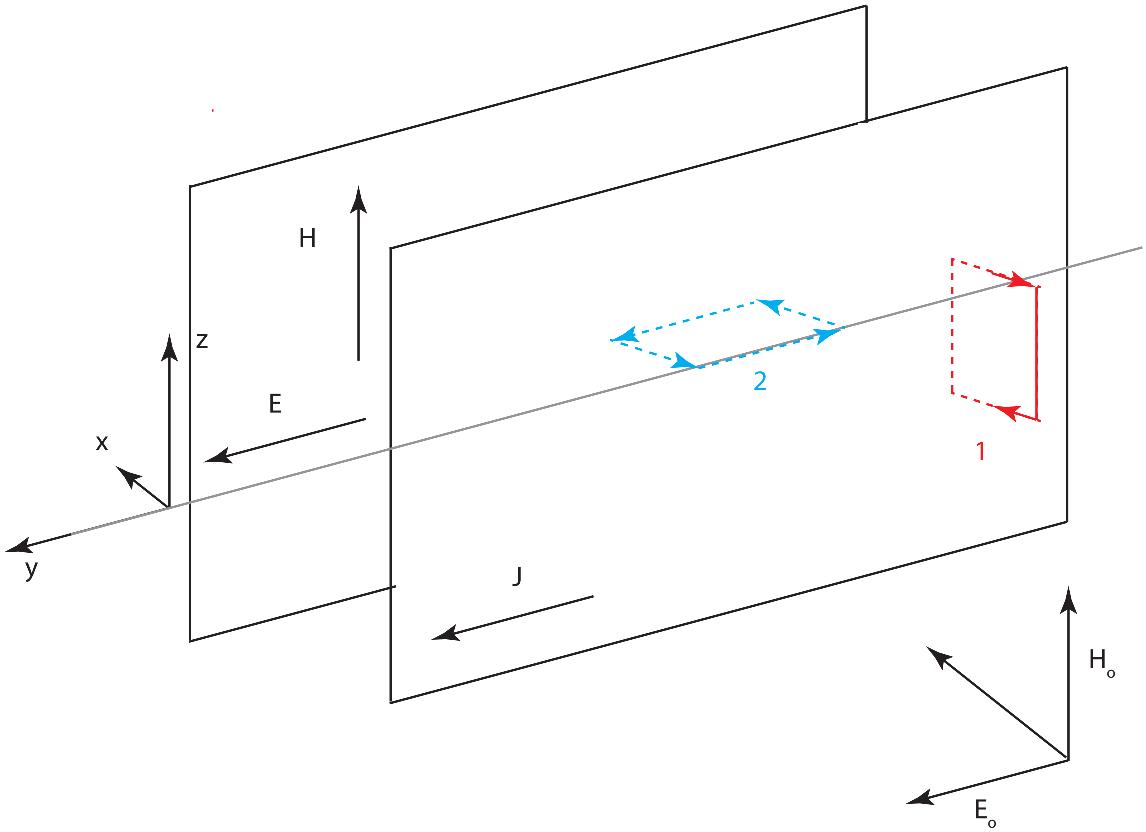

Figure 1: Parallel conducting plates with the loops used in the following calculation.

5 Case of box in a time varying magnetic field

Ref. [1, p. 13] considers the case of a 15 cm copper box in a time varying magnetic field between two Helmholtz coils. The box was made from PC board with 36 mu copper, $\sigma = 6 \times 10^7$ S/m. For these values, $\delta = l$ when $\nu = 3.3 \times 10^6$ MHz. The uniform magnetic field setup by the Helmholtz coils ensures there is no displacement field since $\vec{\nabla} \times \vec{H} = 0$, so $E_\circ = 0$ and

$$\frac{H}{H_o} = \frac{1}{1 + \mu_\circ \omega \sigma X l/2} = \frac{1}{1 + Xl/\delta^2}.$$

The attenuation from the skin depth is $\exp -l/\delta \sim 1 - l/\delta \sim 1/1 + l/\delta$. Comparing with the expression above, an effective skin depth can be defined as

$$\delta_{\text{eff}} = \frac{\delta^2}{X}.$$

Figure 2 does the attenuation from surface currents and skin depth compared with measurements from [1].

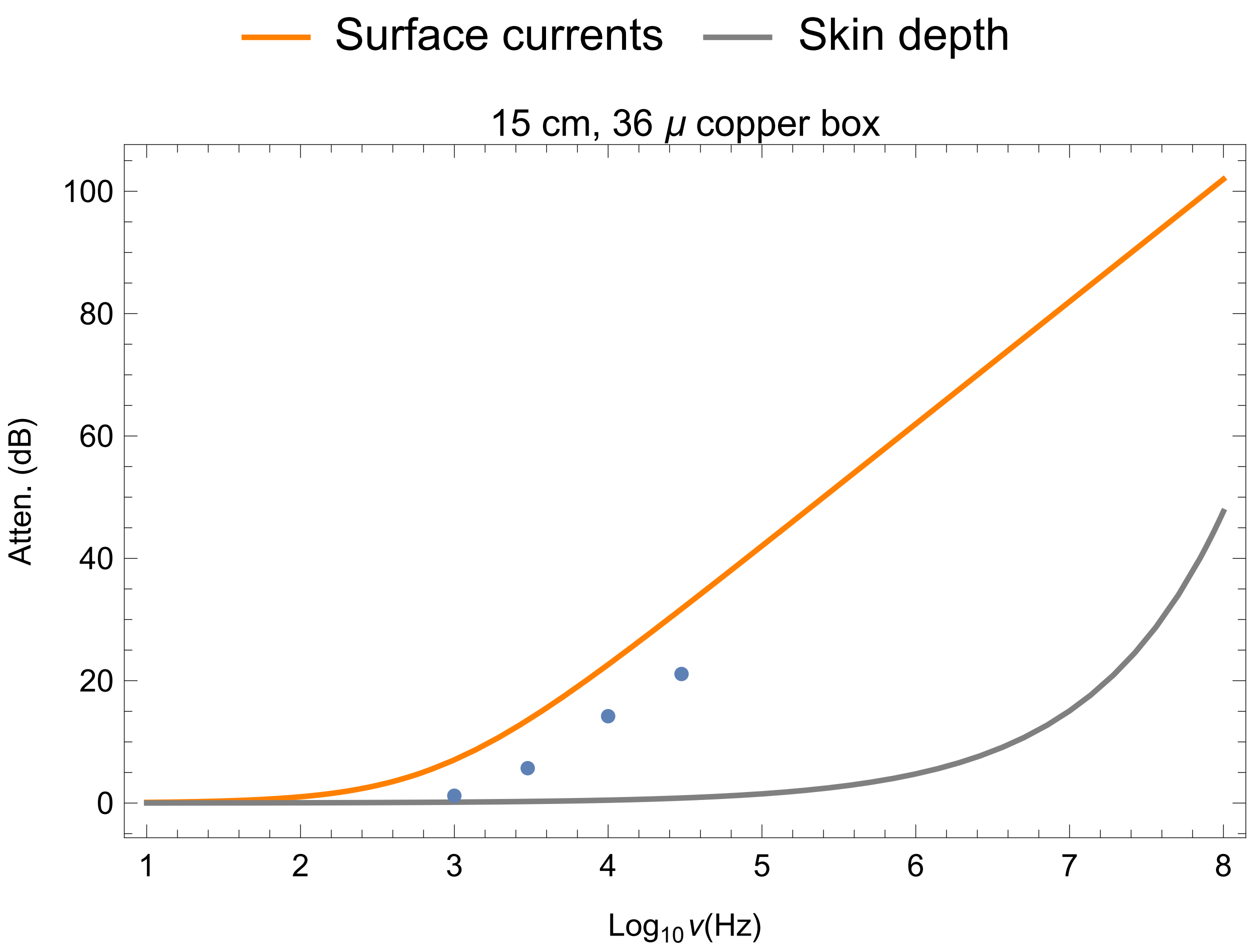

Figure 2: Attenuation from surface currents and skin depth compared with measurements (blue points) from [1].

References

[1] P. Horowitz and W. Hill. *The Art of Electronics: The X Chapters*. Cambridge University Press, 2020.