The First Three Minutes
Meeting 8

Peter Fisher

March 1, 2021

Meeting 8 – The First Three Minutes

- Announcements
- 30,000 ft
- Chapter VII
- **The Weak and Strong Interaction in the early universe**
- **Break**
- The Standard Model of Particle Physics

Announcements

- Notes, slides, etc. on website, tinyurl.com/firstthreeminutes
- Please read Chapter VIII and Afterword for next week
- Questions

30,000’ view

Galaxies are the “atoms” of the universe

When viewed on the 100 Mly scale, the universe is uniform and isotropic

Hubble’s redshift measurements showed all the galaxies are moving away from us. Their recession speed is proportional to how far away they are. H_o is the proportionality constant.

30,000’ (cont.)

The recession of the galaxies led to the idea that the space of the universe is expanding. The expansion is the same everywhere.

The numerical value of H_o implies the universe is 13.7 Gy old.

Penzias and Wilson’s observation of 2.7 K radiation led to the conclusion that neutral hydrogen formed from a plasma 377,000 y after the start of the universe.

30,000’ (cont.)

At 0.01 s, the recipe for a hot universe consists of

- Zero net charge
- Protons, neutrons, electrons at the 1 ppb level compared with photons (and neutrinos)
- T=100 B kelvin for black body photons
- Expansion as $t^{1/2}$

Synthesis of H, D, and He began at 0.01s as the proton-neutron imbalance developed, but was delayed to 3 min by the low binding energy and fragility of the deuteron.

Chapter VII – The First Hunredth of a Second

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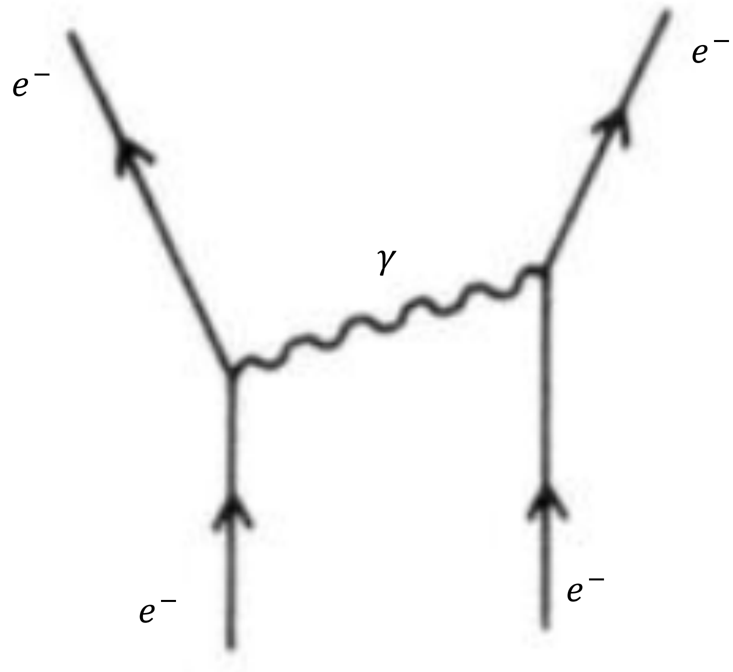
- Expansion as $t^{1/2}$

Before 0.01s, the universe was dominated by the strong interaction

The Language of Particle Physics – Feynman Diagrams



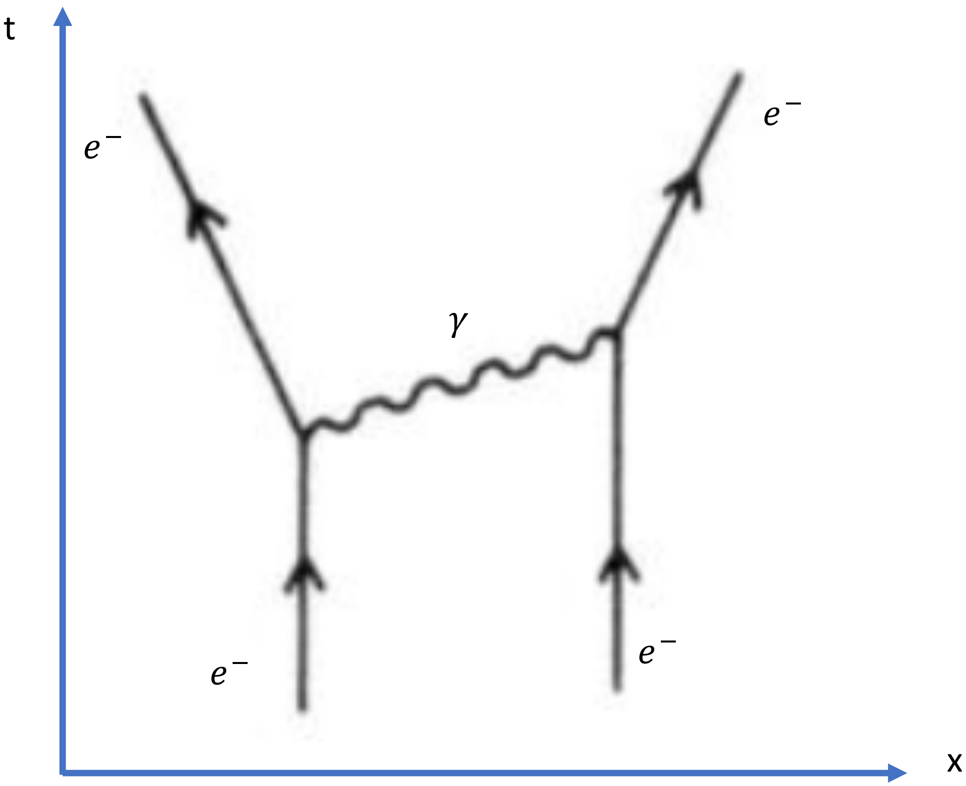
$$e^- + e^- \rightarrow e^- + e^-$$

Quantum Field Theory – Dirac, Feynman, Schwinger, Tomonaga, Dyson, 1940's

Enabled very complex calculations

Feynman developed graphical technique that rendered theory accessible to many.

The Language of Particle Physics – Feynman Diagrams



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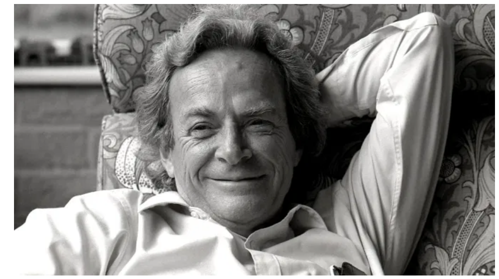
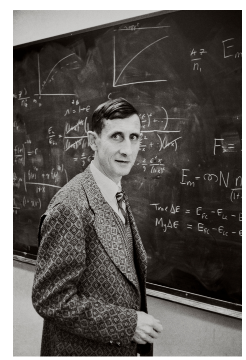
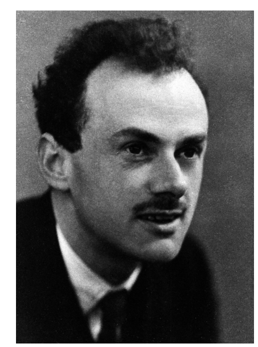






The Language of Particle Physics – Feynman Diagrams

$$e^- + e^- \rightarrow e^- + e^-$$

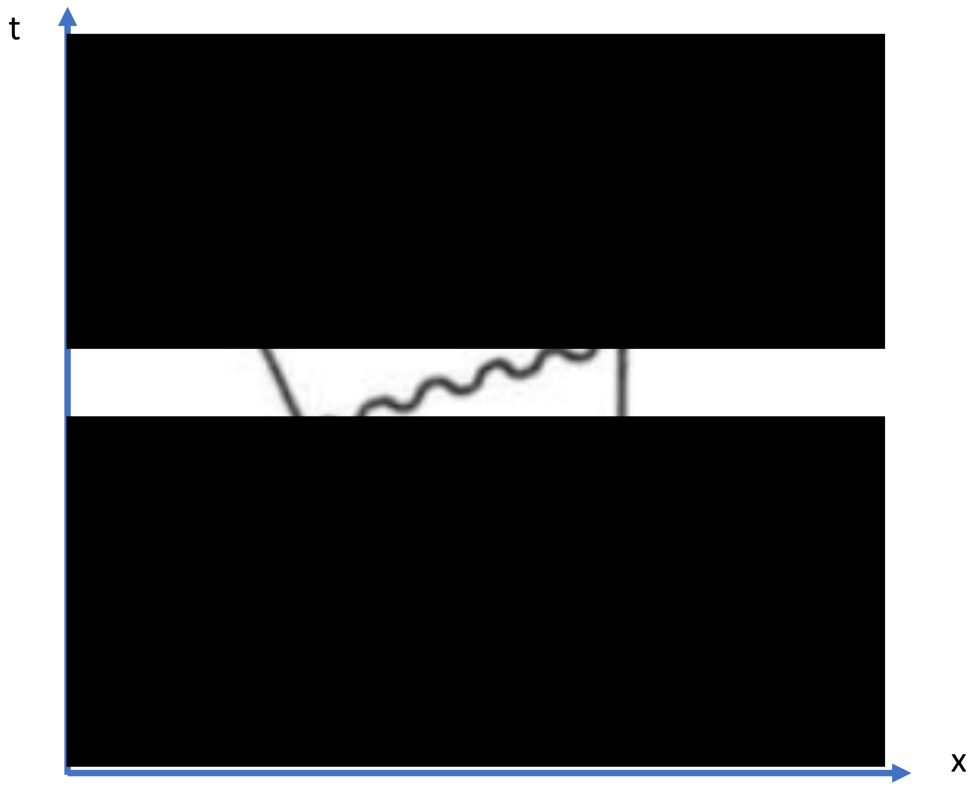
At start, two electrons sitting stationary near each other

The Language of Particle Physics – Feynman Diagrams

$$e^- + e^- \rightarrow e^- + e^-$$

Later, one electron emits a photon and deflects. The photon travels toward the other electron

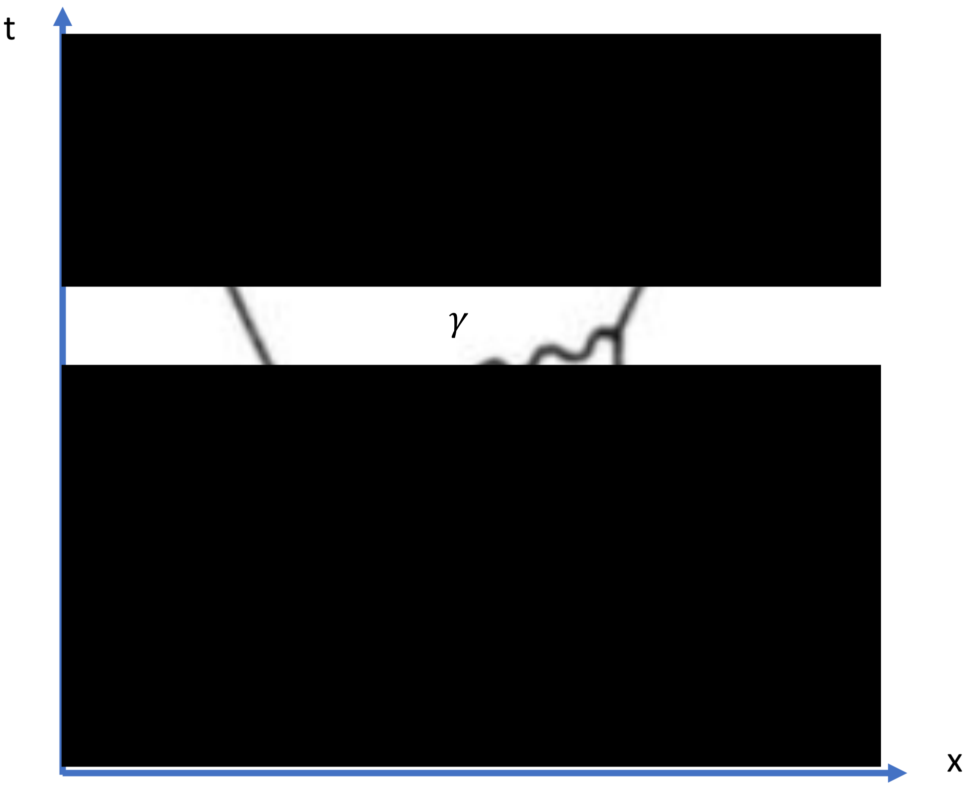
The Language of Particle Physics – Feynman Diagrams



$$e^- + e^- \rightarrow e^- + e^-$$

The photon travels through space. The first electron continues to recoil the other electron remains at rest.

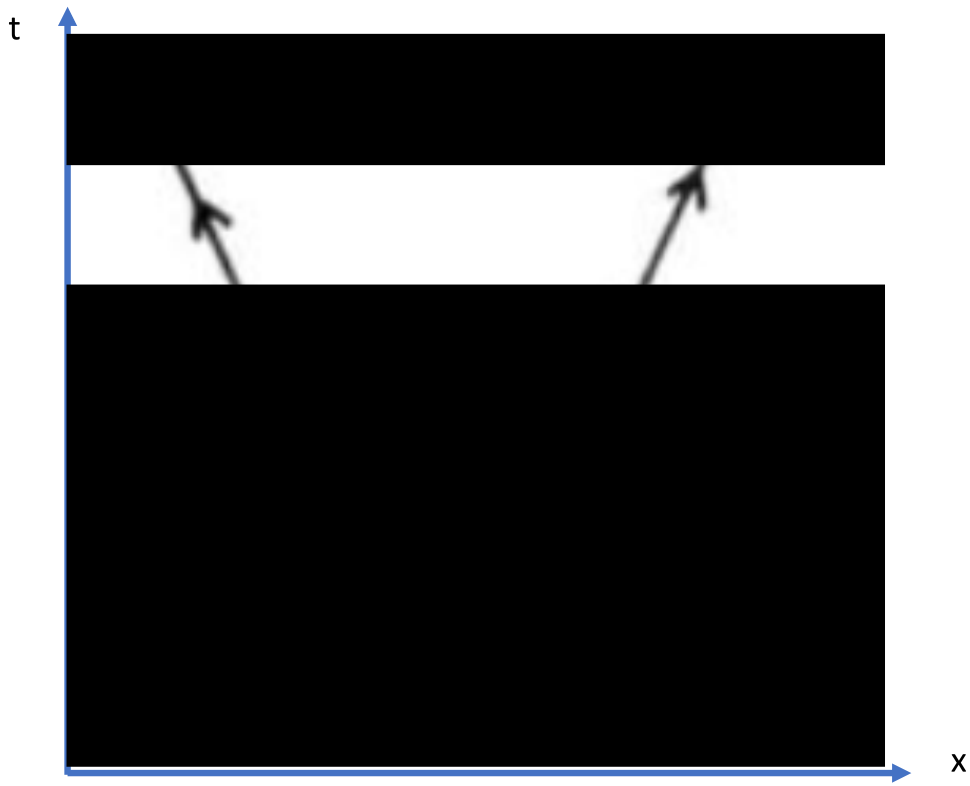
The Language of Particle Physics – Feynman Diagrams



$$e^- + e^- \rightarrow e^- + e^-$$

The second electron absorbs the photon and begin to move to the right. The first electron continues to move to the left.

The Language of Particle Physics – Feynman Diagrams



$$e^- + e^- \rightarrow e^- + e^-$$

The two electrons more away from each other.

tude for each space-time path available.[1] In view of the fact that in classical physics positrons could be viewed as electrons proceeding along world lines toward the past (reference 7) the attempt was made to remove, in the relativistic case, the restriction that the paths must proceed always in one direction in time. It was discovered that the results could be even more easily understood from a more familiar physical viewpoint, that of scattered waves. This viewpoint is the one used in this paper. After the equations were worked out physically the proof of the equivalence to the second quantization theory was found.[2]

First we discuss the relation of the Hamiltonian differential equation to its solution, using for an example the Schrödinger equation. Next we deal in an analogous way with the Dirac equation and show how the solutions may be interpreted to apply to positrons. The interpretation seems not to be consistent unless the electrons obey the exclusion principle. (Charges obeying the Klein-Gordon equations can be described in an analogous manner, but here consistency apparently requires Bose statistics.)[3] A representation in momentum and energy variables which is useful for the calculation of matrix elements is described. A proof of the equivalence of the method to the theory of holes in second quantization is given in the Appendix.

2. GREEN'S FUNCTION TREATMENT OF SCHRÖDINGER'S EQUATION

We begin by a brief discussion of the relation of the non-relativistic wave equation to its solution. The ideas will then be extended to relativistic particles, satisfying Dirac's equation, and finally in the succeeding paper to interacting relativistic particles, that is, quantum electrodynamics.

The Schrödinger equation

$$i\partial\psi/\partial t = H\psi, \tag{1}$$

describes the change in the wave function ψ in an infinitesimal time Δt as due to the operation of an operator $\exp(-iH\Delta t)$. One can ask also, if $\psi(\mathbf{x}_1, t_1)$ is the wave function at $\mathbf{x}_1$ at time t_1, what is the wave function at time $t_2 > t_1$? It can always be written as

$$\psi(\mathbf{x}_2, t_2) = \int K(\mathbf{x}_2, t_2; \mathbf{x}_1, t_1)\psi(\mathbf{x}_1, t_1)d^3\mathbf{x}_1, \tag{2}$$

where K is a Green's function for the linear Eq. (1). (We have limited ourselves to a single particle of coordinate $\mathbf{x}$, but the equations are obviously of greater generality.) If H is a constant operator having eigenvalues E_n, eigenfunctions ϕ_n so that $\psi(\mathbf{x}, t_1)$ can be expanded as $\sum_n C_n\phi_n(\mathbf{x})$, then $\psi(\mathbf{x}, t_2) = \exp(-iE_n(t_2-t_1)) \times C_n\phi_n(\mathbf{x})$. Since $C_n = \int\phi_n^*(\mathbf{x}_1)\psi(\mathbf{x}_1, t_1)d^3\mathbf{x}_1$, one finds (where we write 1 for $\mathbf{x}_1, t_1$ and 2 for $\mathbf{x}_2, t_2$) in this case

$$K(2,1) = \sum_n \phi_n(\mathbf{x}_2)\phi_n^*(\mathbf{x}_1)\exp(-iE_n(t_2-t_1)), \tag{3}$$

for $t_2 > t_1$. We shall find it convenient for $t_2 < t_1$ to define $K(2,1) = 0$ (Eq. (2) is then not valid for $t_2 < t_1$). It is then readily shown that in general K can be defined by that solution of

$$(i\partial/\partial t_2 - H_2)K(2,1) = i\delta(2,1), \tag{4}$$

which is zero for $t_2 < t_1$, where $\delta(2,1) = \delta(t_2-t_1)\delta(x_2-x_1) \times\delta(y_2-y_1)\delta(z_2-z_1)$ and the subscript 2 on H_2 means that the operator acts on the variables of 2 of $K(2,1)$. When H is not constant, (2) and (4) are valid but K is less easy to evaluate than (3).[4]

We can call $K(2,1)$ the total amplitude for arrival at $\mathbf{x}_2, t_2$ starting from $\mathbf{x}_1, t_1$. (It results from adding an amplitude, $\exp iS$, for each space time path between these points, where S is the action along the path.[1]) The transition amplitude for finding a particle in state $\chi(\mathbf{x}_2, t_2)$ at time t_2, if at t_1 it was in $\psi(\mathbf{x}_1, t_1)$, is

$$\int \chi^*(2)K(2,1)\psi(1)d^3\mathbf{x}_1d^3\mathbf{x}_2. \tag{5}$$

A quantum mechanical system is described equally well by specifying the function K, or by specifying the Hamiltonian H from which it results. For some purposes the specification in terms of K is easier to use and visualize. We desire eventually to discuss quantum electrodynamics from this point of view.

To gain a greater familiarity with the K function and the point of view it suggests, we consider a simple perturbation problem. Imagine we have a particle in a weak potential $U(\mathbf{x}, t)$, a function of position and time. We wish to calculate $K(2,1)$ if U differs from zero only for t between t_1 and t_2. We shall expand K in increasing powers of U:

$$K(2,1) = K_0(2,1) + K^{(1)}(2,1) + K^{(2)}(2,1) + \cdots. \tag{6}$$

To zero order in U, K is that for a free particle, $K_0(2,1)$.[4] To study the first order correction $K^{(1)}(2,1)$, first consider the case that U differs from zero only for the infinitesimal time interval Δt_3 between some time t_3 and $t_3+\Delta t_3$ $(t_1 < t_3 < t_2)$. Then if $\psi(1)$ is the wave function at $\mathbf{x}_1, t_1$, the wave function at $\mathbf{x}_3, t_3$ is

$$\psi(3) = \int K_0(3,1)\psi(1)d^3\mathbf{x}_1, \tag{7}$$

since from t_1 to t_3 the particle is free. For the short interval Δt_3 we solve (1) as

$$\psi(\mathbf{x}, t_3+\Delta t_3) = \exp(-iH\Delta t_3)\psi(\mathbf{x}, t_3) = (1 - iH_0\Delta t_3 - iU\Delta t_3)\psi(\mathbf{x}, t_3),$$

[1] R. P. Feynman, Rev. Mod. Phys. 20, 367 (1948).

[2] The equivalence of the entire procedure (including photon interactions) with the work of Schwinger and Tomonaga has been demonstrated by F. J. Dyson, Phys. Rev. 75, 486 (1949).

[3] These are special examples of the general relation of spin and statistics deduced by W. Pauli, Phys. Rev. 58, 716 (1940).

[4] For a non-relativistic free particle, where $\phi_n = \exp(i\mathbf{p}\cdot\mathbf{x})$, $E_n = \mathbf{p}^2/2m$, (3) gives, as is well known

$$K_0(2,1) = \int \exp[-(i\mathbf{p}\cdot\mathbf{x}_1 - i\mathbf{p}\cdot\mathbf{x}_2) - ip^2(t_2-t_1)/2m]d^3\mathbf{p}(2\pi)^{-3} = (2\pi im^{-1}(t_2-t_1))^{-\frac{3}{2}}\exp(\tfrac{1}{2}im(\mathbf{x}_2-\mathbf{x}_1)^2(t_2-t_1)^{-1})$$

for $t_2 > t_1$, and $K_0 = 0$ for $t_2 < t_1$.

conjugate matrix. For the particular representation in which all elements of γ_4 are imaginary, while all elements of the other matrices are real, the conditions on C are satisfied with $C = -\gamma_4$. With this choice, $\psi'(x) = \psi^*(x)$; charge and Hermitian conjugation are equivalent. Finally,

$$\kappa_0 = m_0c/\hbar, \tag{1.7}$$

where m_0 is the mechanical proper mass of the electron.

The equations of motion of the coupled electromagnetic and electron-positron matter fields can be derived from the variational principle:

$$\delta\int \mathcal{L}d\omega = 0, \tag{1.8}$$

where the Lagrangian density $\mathcal{L}$ is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\frac{\partial A_\nu(x)}{\partial x_\mu}\frac{\partial A_\nu(x)}{\partial x_\mu}\\ &-\frac{\hbar c}{2}\bar\psi(x)\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu}-\frac{ie}{\hbar c}A_\mu(x)\right)+\kappa_0\right]\psi(x)\\ &-\frac{\hbar c}{2}\bar\psi'(x)\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu}+\frac{ie}{\hbar c}A_\mu(x)\right)+\kappa_0\right]\psi'(x),\end{aligned} \tag{1.9}$$

and is so constructed that it is invariant with respect to Lorentz transformations, gauge transformations and charge conjugation. The proof of Lorentz invariance follows the conventional treatment and need not be repeated. Gauge invariance, that is, invariance under the combined transformations

$$\begin{aligned}A_\mu(x) &\to A_\mu(x) - \frac{\partial\Lambda(x)}{\partial x_\mu}\\ \psi(x) &\to \exp\left[-\frac{ie}{\hbar c}\Lambda(x)\right]\psi(x)\\ \psi'(x) &\to \exp\left[\frac{ie}{\hbar c}\Lambda(x)\right]\psi'(x)\end{aligned} \tag{1.10}$$

induced by a scalar function of position, $\Lambda(x)$, would be generally valid were it not for the term in the Lagrangian density that refers to the electromagnetic field alone. The addition to $\mathcal{L}$ arising therefrom is

$$-\frac{\partial}{\partial x_\mu}\left[\left(A_\nu+\frac{1}{2}\frac{\partial\Lambda}{\partial x_\nu}\right)\frac{\partial^2\Lambda}{\partial x_\mu\partial x_\nu}\right] + \left(A_\nu+\frac{1}{2}\frac{\partial\Lambda}{\partial x_\nu}\right)\frac{\partial}{\partial x_\nu}\frac{\partial^2\Lambda}{\partial x_\mu^2},$$

of which the first term has no effect on the equations of motion. Hence gauge invariance is restricted to the group of generating functions that obey

$$\frac{\partial^2\Lambda(x)}{\partial x_\mu^2} = \Box^2\Lambda(x) = 0. \tag{1.11}$$

Invariance under charge conjugation expresses the complete symmetry between positive and negative charge. The interchange of $\psi(x)$ and $\psi'(x)$, together with $+e$ and $-e$, evidently leaves the Lagrangian density unaltered.

In order to obtain the equations of motion for the matter field, it is necessary to express the Lagrangian density entirely in terms of $\psi(x)$ and $\bar\psi(x)$, or alternatively, $\psi'(x)$ and $\bar\psi'(x)$. By virtue of Eqs. (1.3), (1.4), and (1.5), the following relations hold

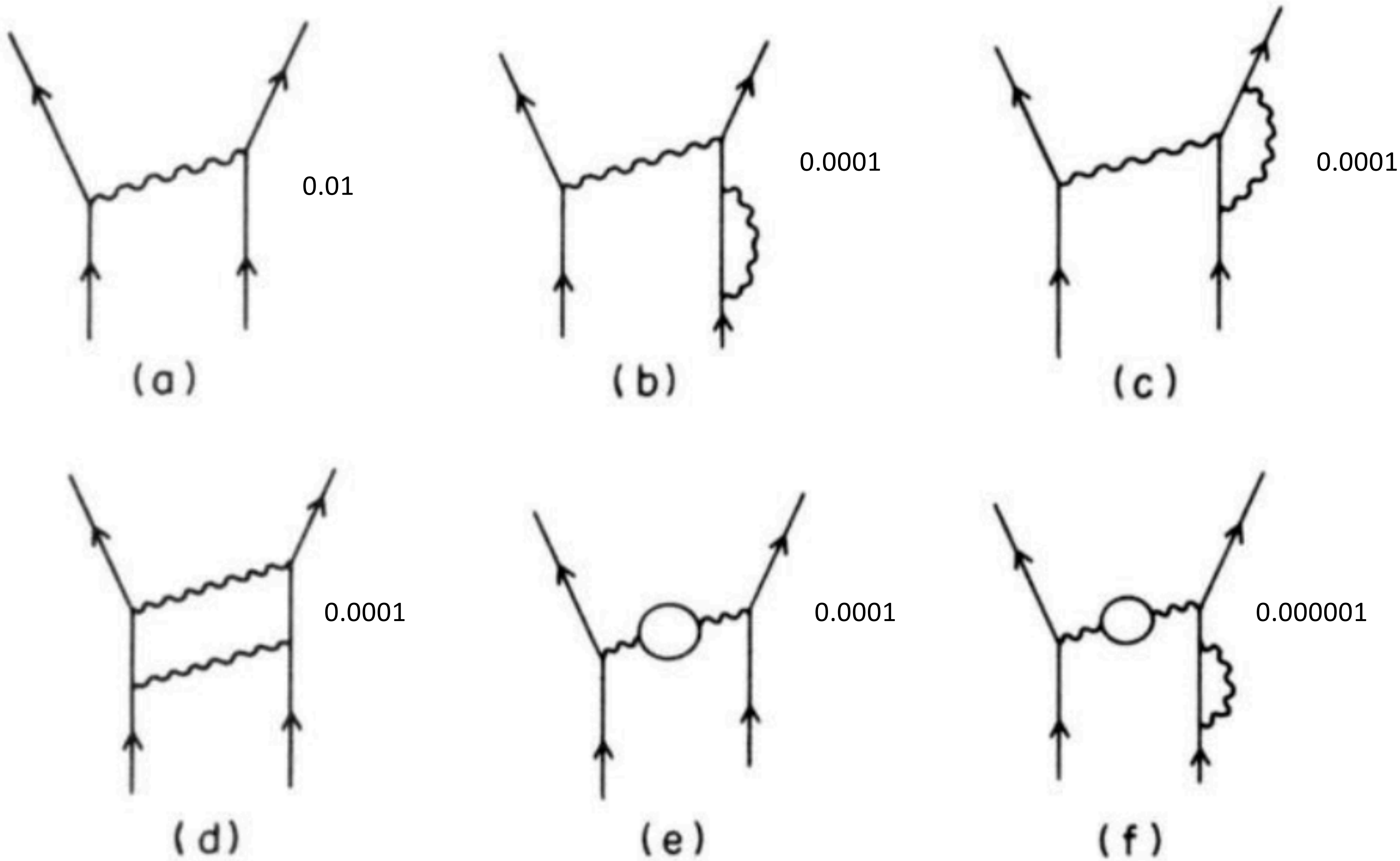
$$\begin{aligned}\bar\psi'\gamma_\mu\psi' &= \psi C^{-1T}\gamma_\mu C\bar\psi = \psi\gamma_\mu^T\bar\psi\\ \bar\psi'\psi' &= \psi C^{-1T}C\bar\psi = -\psi\bar\psi,\end{aligned} \tag{1.12}$$

and therefore the third term of (1.9) can be written

$$-\frac{\hbar c}{2}\psi(x)\left[\gamma_\mu^T\left(\frac{\partial}{\partial x_\mu}+\frac{ie}{\hbar c}A_\mu(x)\right)-\kappa_0\right]\bar\psi(x).$$

We find, as the result of variation, apart from discarded divergences,

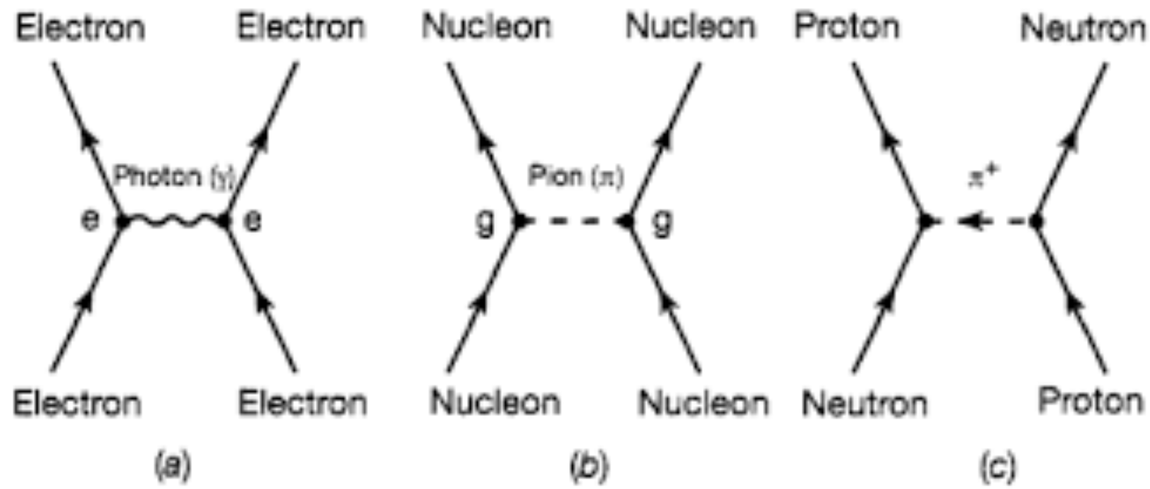
$$\begin{aligned}\delta\mathcal{L} = &\frac{1}{2}\delta A_\mu\left[\Box^2A_\mu+\frac{1}{c}j_\mu\right]+\frac{1}{2}\left[\Box^2A_\mu+\frac{1}{c}j_\mu\right]\delta A_\mu\\ &-\frac{\hbar c}{2}\delta\bar\psi\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu}-\frac{ie}{\hbar c}A_\mu\right)+\kappa_0\right]\psi\\ &+\frac{\hbar c}{2}\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu}-\frac{ie}{\hbar c}A_\mu\right)+\kappa_0\right]\psi\delta\bar\psi\\ &-\frac{\hbar c}{2}\delta\psi\left[\gamma_\mu^T\left(\frac{\partial}{\partial x_\mu}+\frac{ie}{\hbar c}A_\mu\right)-\kappa_0\right]\bar\psi\\ &+\frac{\hbar c}{2}\left[\gamma_\mu^T\left(\frac{\partial}{\partial x_\mu}+\frac{ie}{\hbar c}A_\mu\right)-\kappa_0\right]\bar\psi\delta\psi = 0,\end{aligned} \tag{1.13}$$


0.01
(a)
0.0001
(b)
0.0001
(c)
0.0001
(d)
0.0001
(e)
0.000001
(f)

Strong Interactions

Work the same way except

- Charged: π^+, π^0, π^-
- Each photon counts equally
- Other mesons carry the strong force



Pions hold the nucleus together in the same way that photons hold atoms together.

Pions in the early universe

Pion threshold is 1.5T K, below threshold at 1.2 μs

Pions are very unstable ($\pi^{\pm}$: 26 ns, π^{0}: $\mathbf{0.08}$ fs), disappear almost instantly

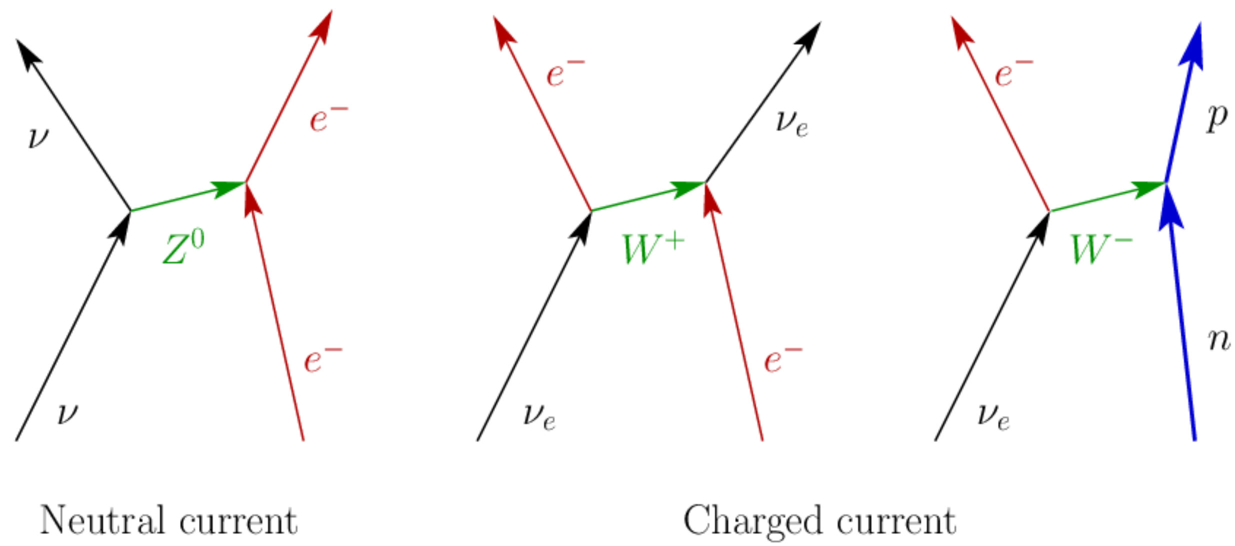
Before n,p threshold, a plasma of n,p, and pions. Interactions are so strong, then medium is like a soup. Impossible to compute anything -- not separatable into collisions.

Weak Interactions

Work the same way!

Except

- W^+, W^-, Z^0
- Very massive -> very short range, very weak



W and Z particles were predicted by Weinberg et al. At masses of about 100 proton masses. Observed in 1983 at CERN.

Electroweak phase transition

Above T=3,000,000 B K, t=11 ns, the thermal energy of the particles did not matter and the weak and electromagnetic interactions “looked” exactly the same.

As temperature dropped, the interactions involving the W and Z particles slowed down and stopped, leaving only the EM.

This was the phase transition.

Timeline

Earlier than 11 ns	Weak and EM interactions equally strong, universe a soupy plasma of all particles
11 ns – 1 ms	Weak interactions, slow down and EM continues. Universe a soupy plasma all particles
1ms – 0.01 s	Pions disappear, neutrons and protons annihilate, leaving ppb level
0.01 s	Stage set for nucleosynthesis

5 m Break

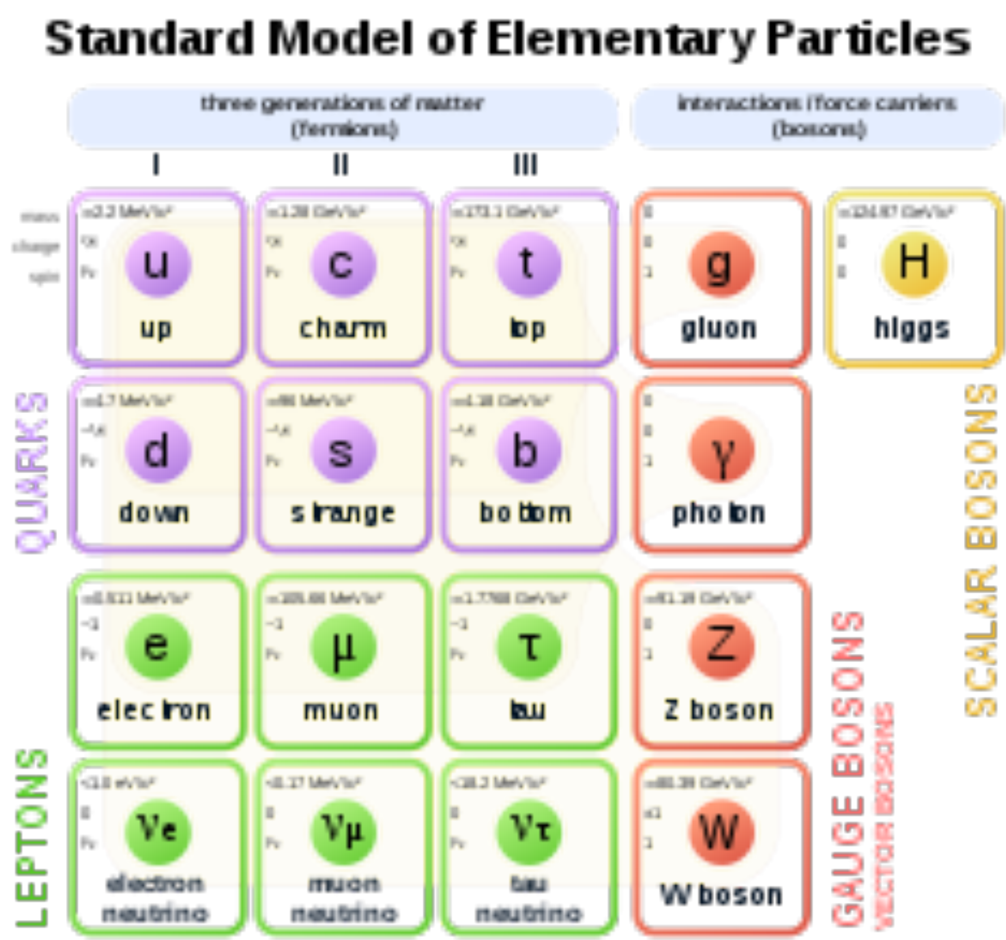
Particles of the Standard Model

Emerged from Weinberg, Glashow, Salam, and others

Matter particles and force carriers

All follow Feynman rules

Enormous predictive power



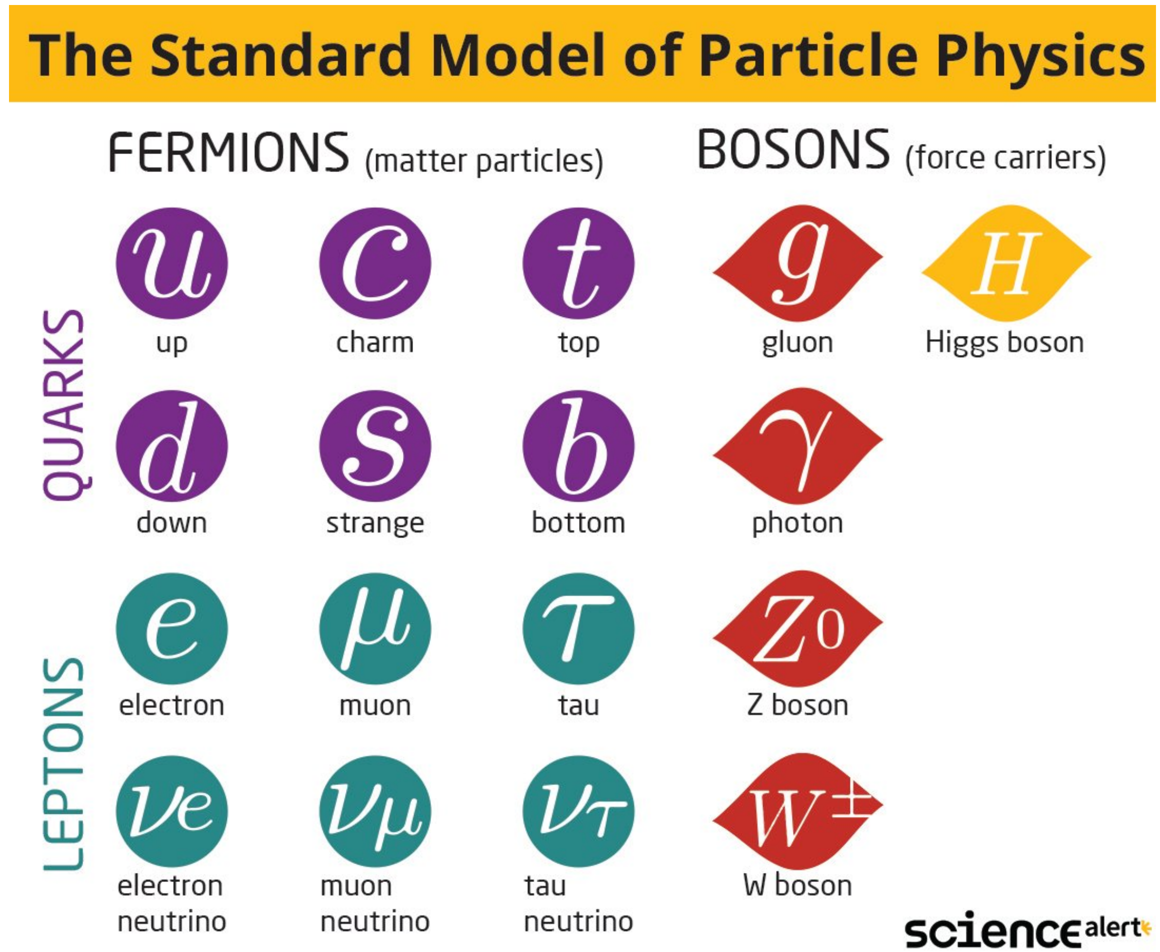
Particles of the Standard Model (cont.)

Does not includes gravity, despite 40 years of effort.

Testing at the ppb level in some ways



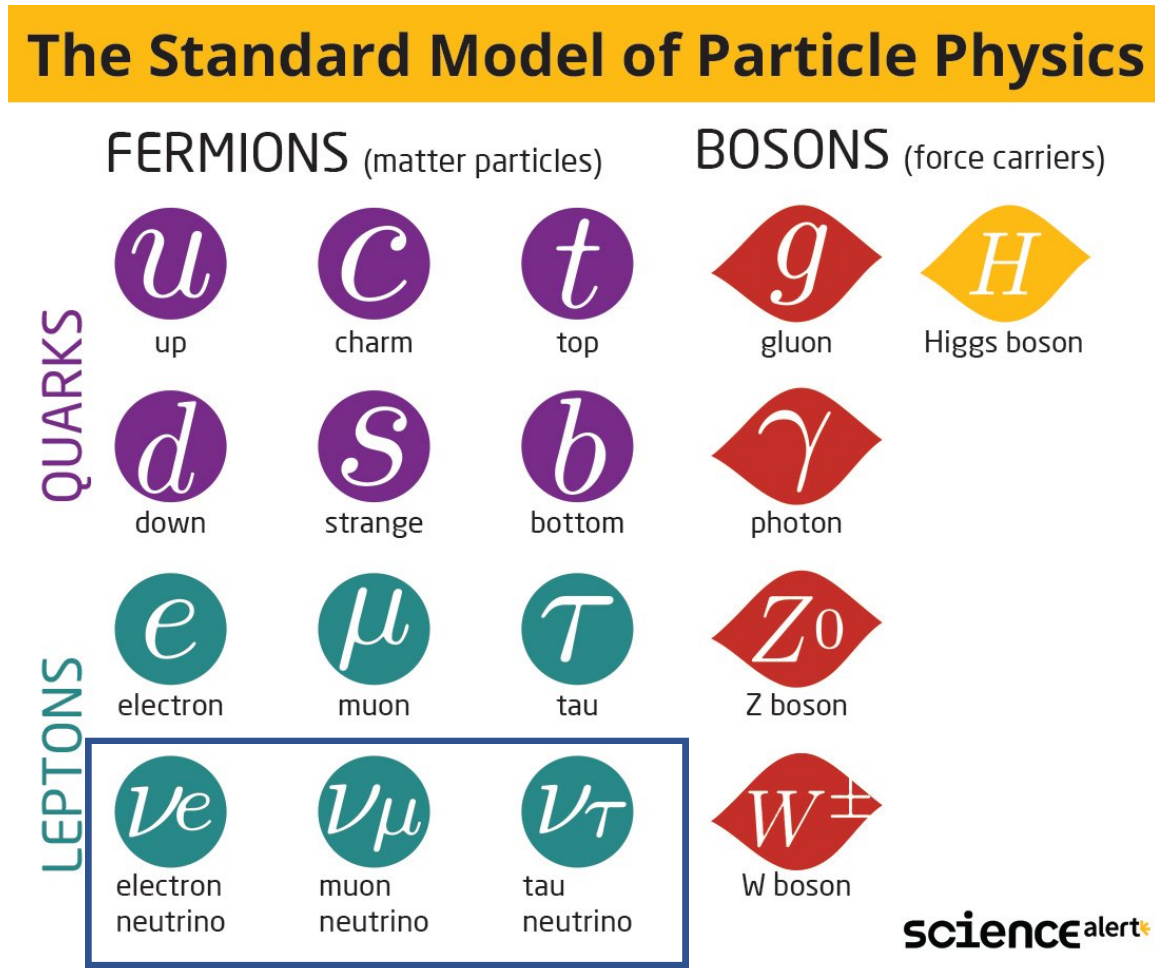

Weinberg, Glashow, Salam, and Gaillard



Particles of the Standard Model (cont.)

Neutrinos

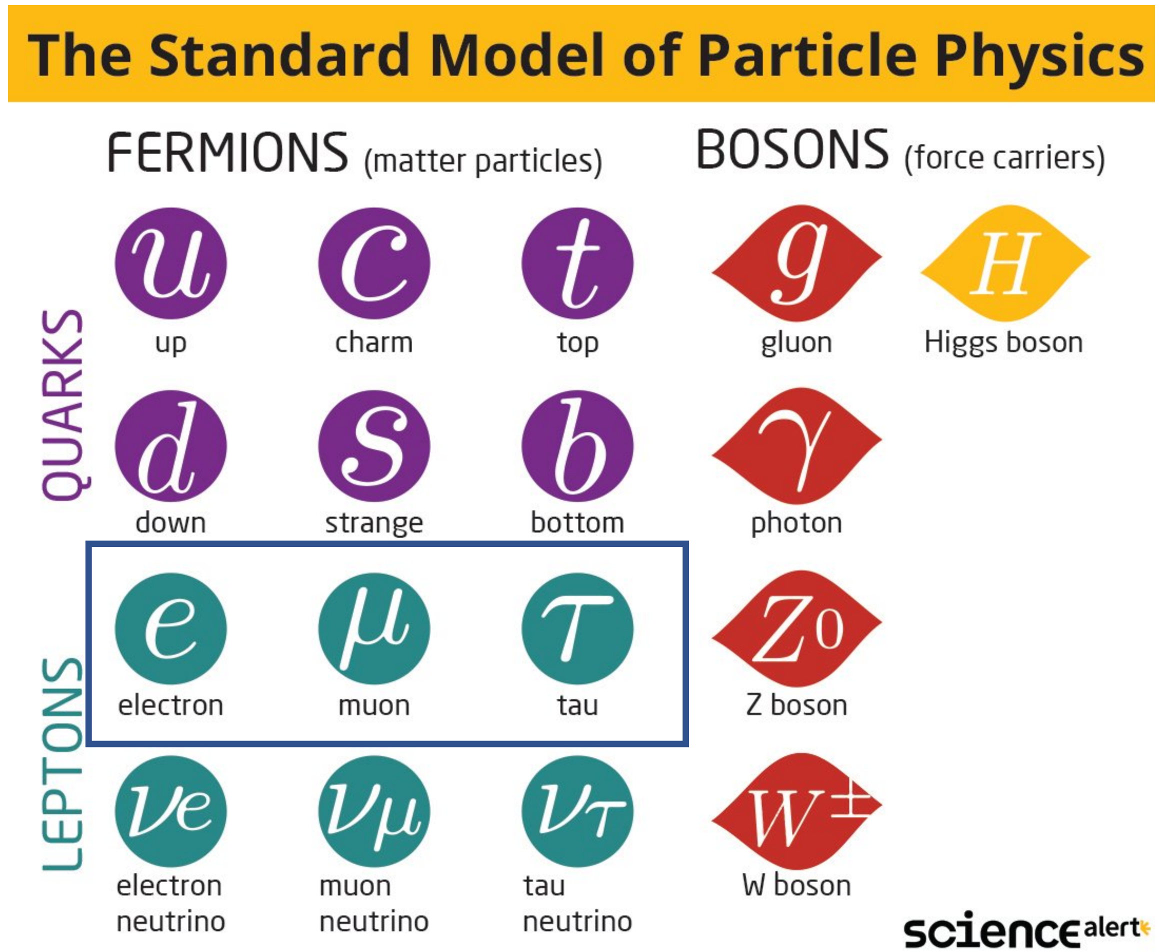
- Weak interactions only
- Mass 1 trillionth of a proton
- Partner with lepton
- “Oscillate”



Particles of the Standard Model (cont.)

Charged leptons

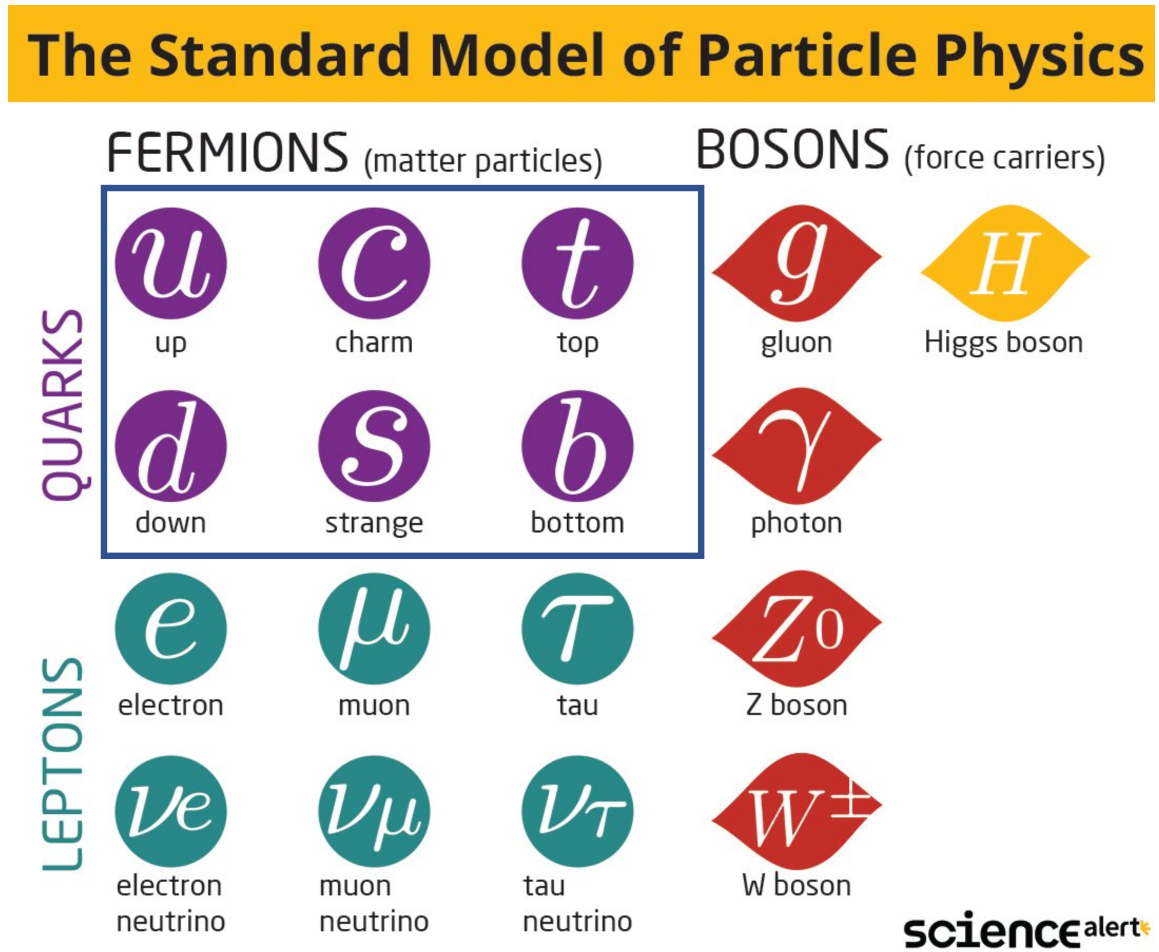
- Weak and EM interactions
- Mass from 1/2000 to 1.8 proton
- Partner with neutrino



Particles of the Standard Model (cont.)

Quarks

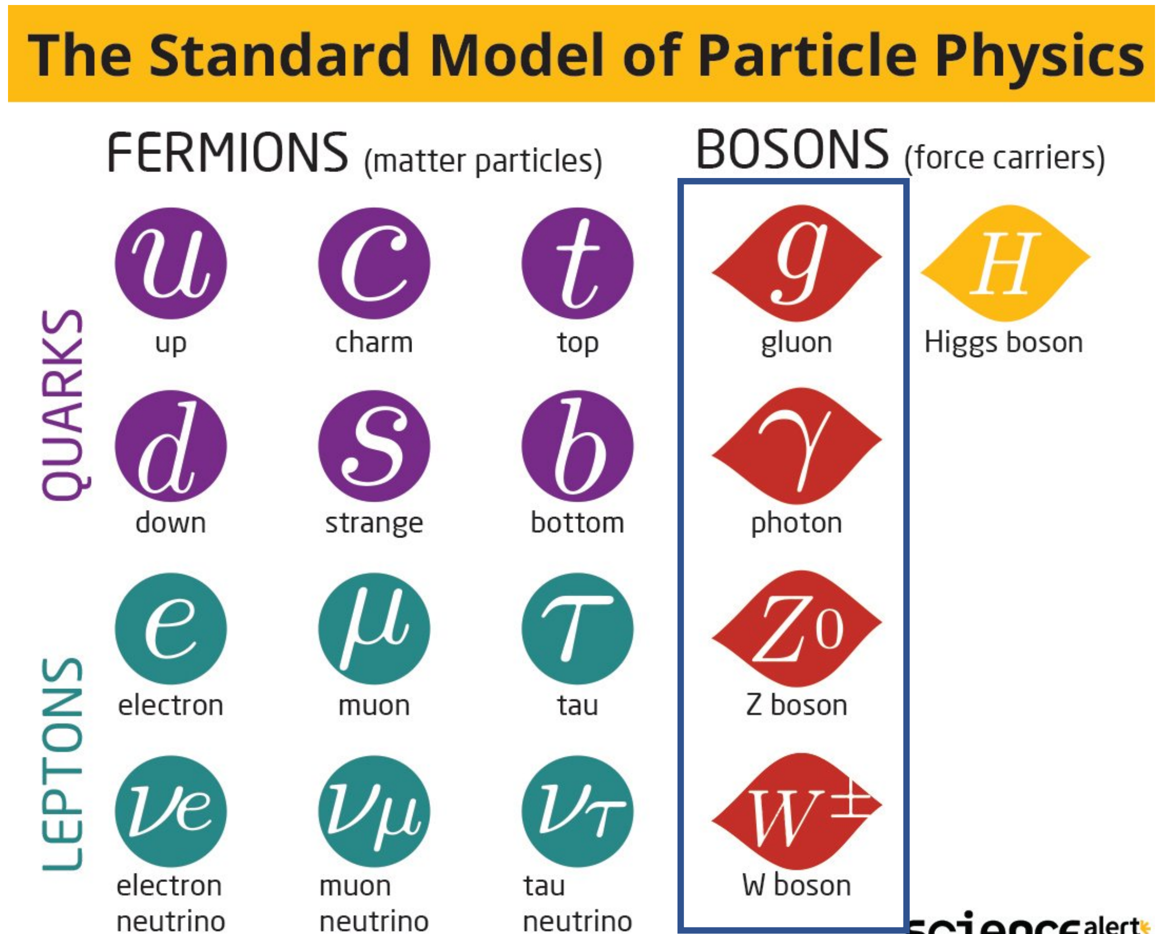
- Weak, EM, strong interactions
- Mass from 1/200 th to 176 proton
- Come in pairs with similar properties
- Bind together to form nuclear particles



Particles of the Standard Model (cont.)

Force carriers

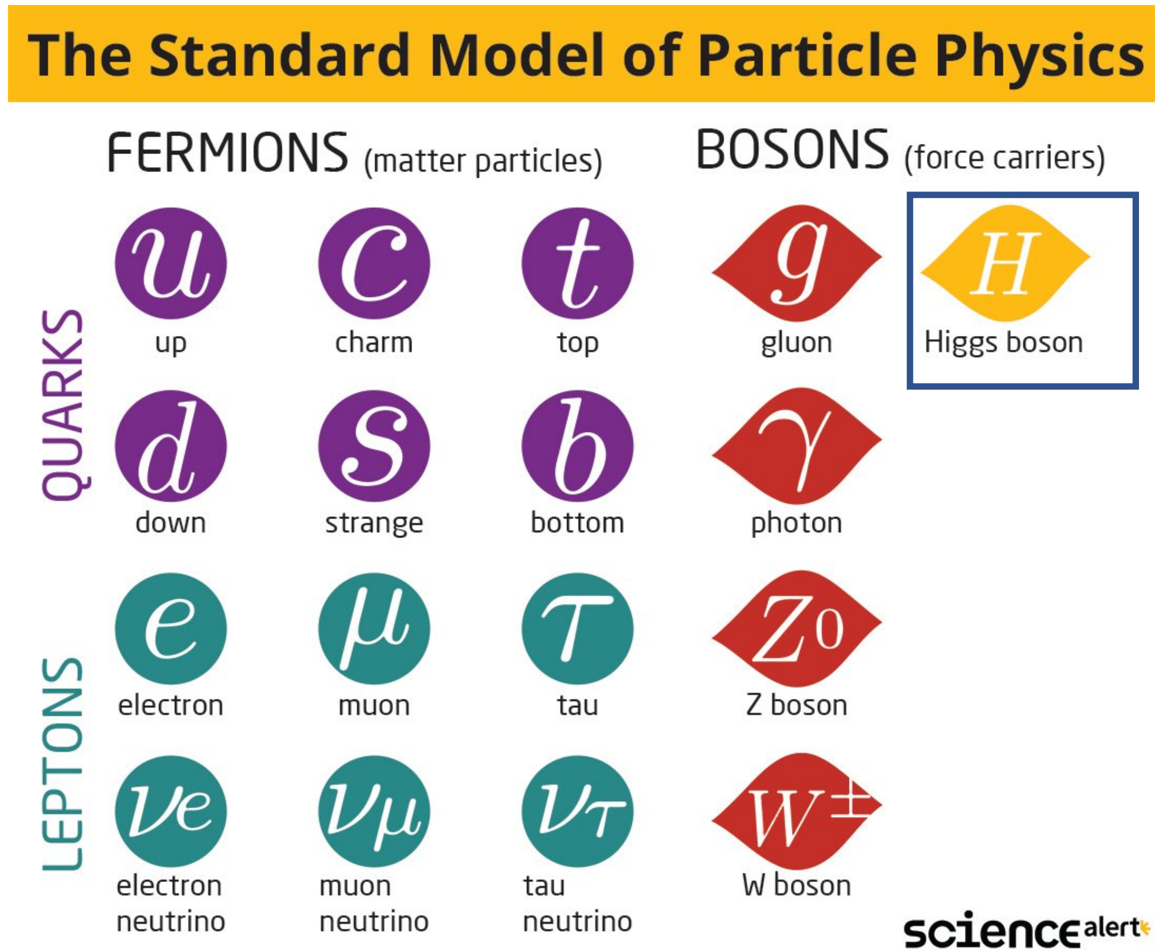
- Photon – massles carrier of EM interaction
- Gluon – massless carrier of the strong interaction - 3 kinds
- W, Z massive, 100 proton, carrier of weak interaction



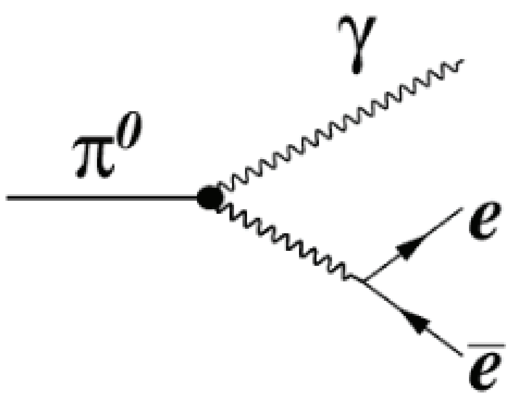
Particles of the Standard Model (cont.)

Higgs boson

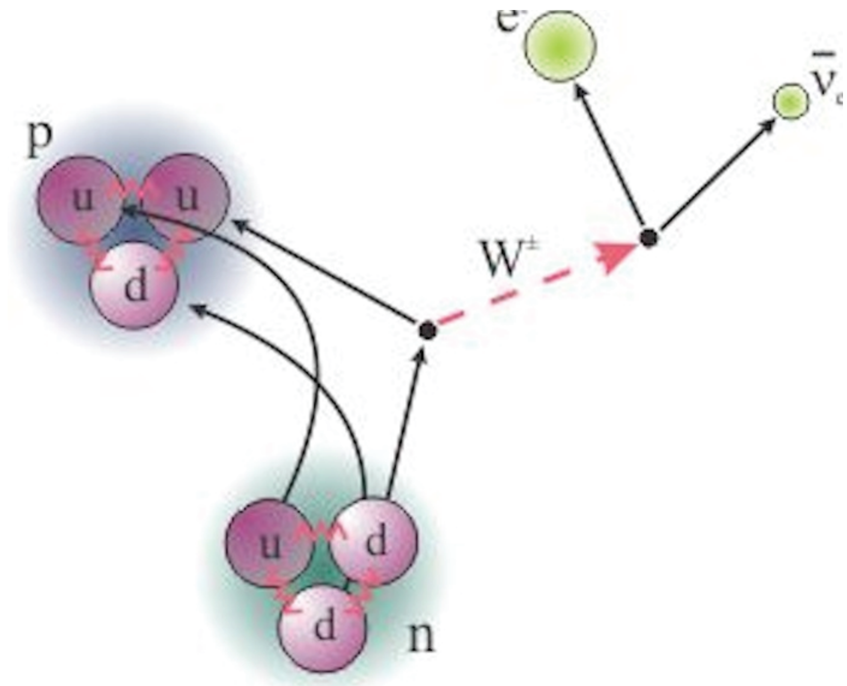
- Field is “everywhere”
- Interaction with the view generates the mass of all the other particles
- 126 proton mass



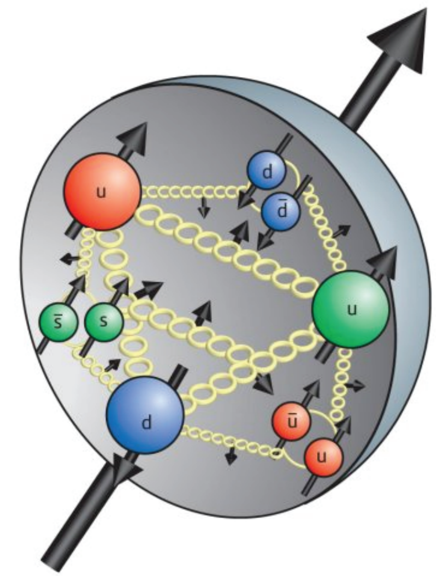
Interactions



Electromagnetic decay

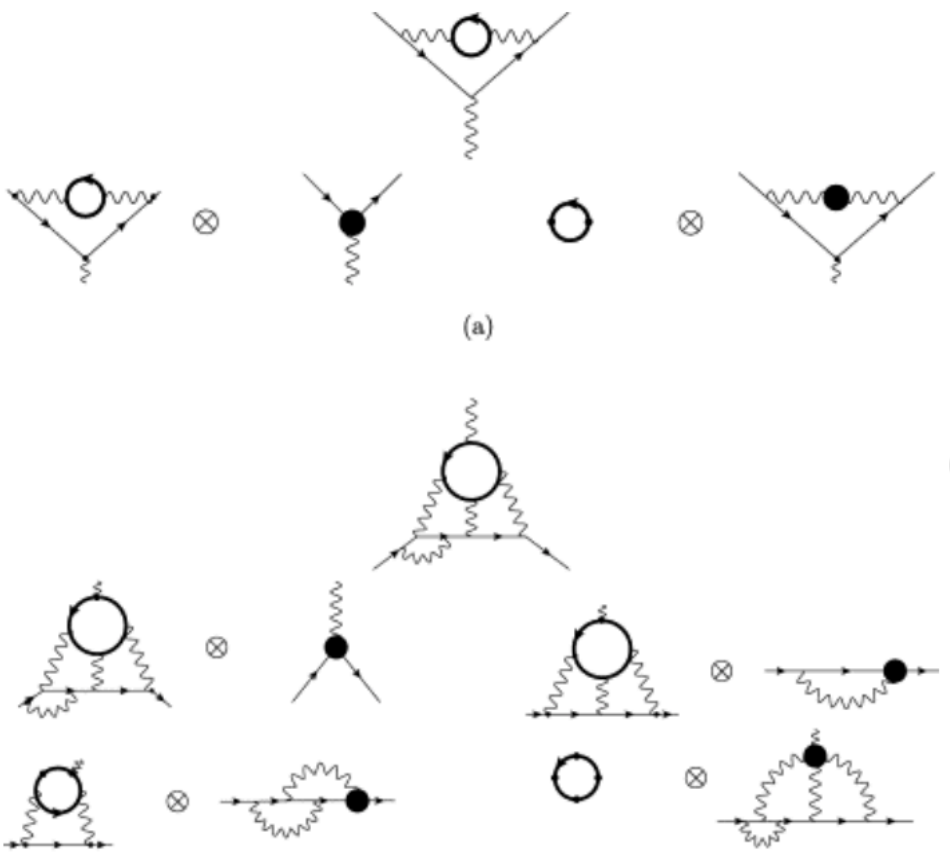


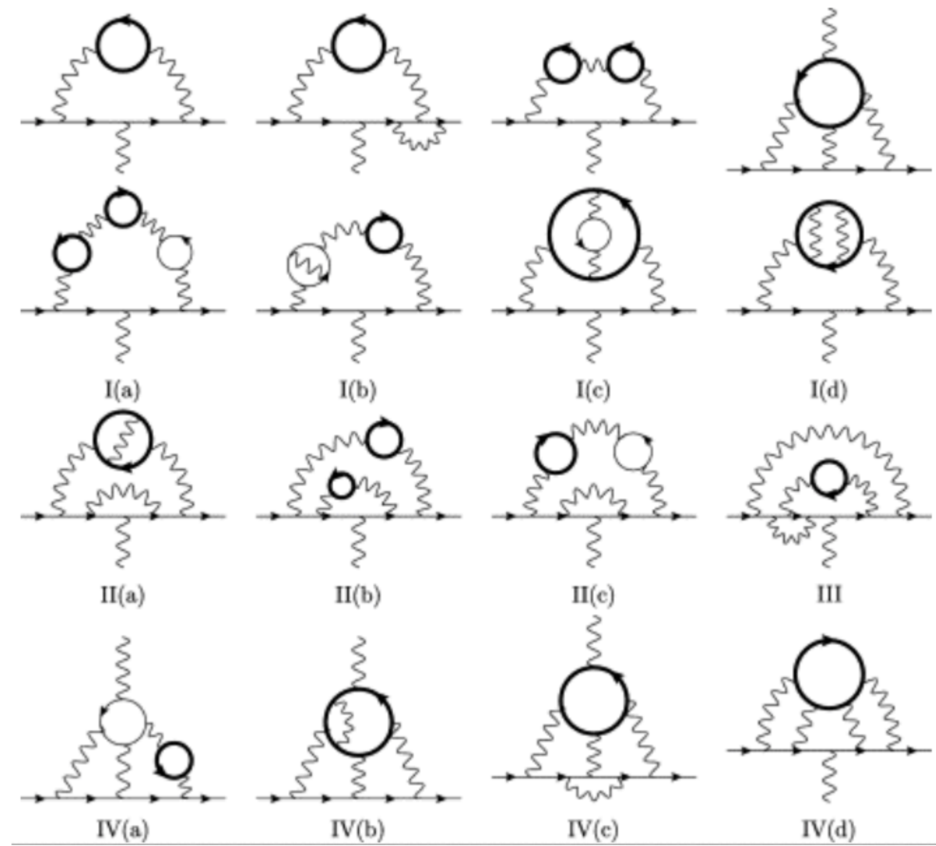
Weak decay



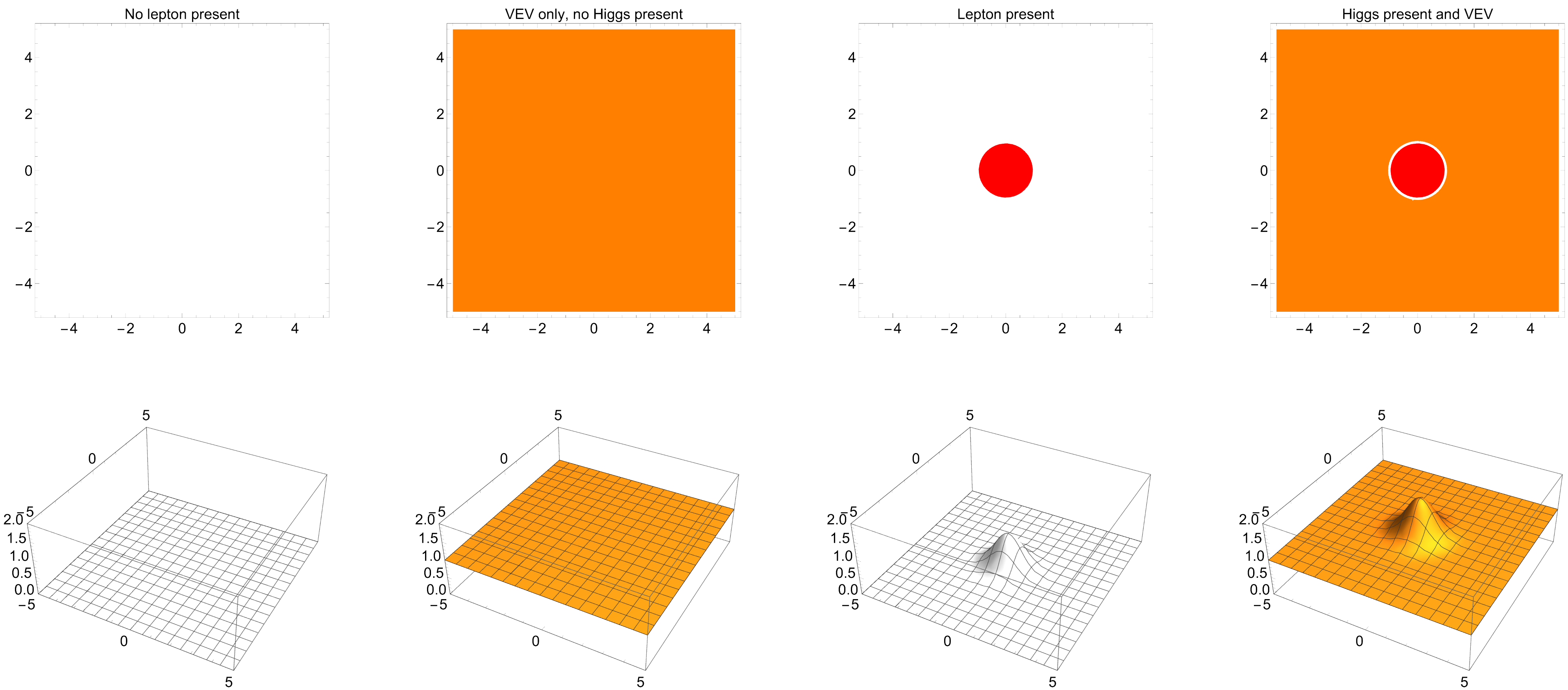
Strong interactions

Diagrams contributing to the magnetism of the muon





Higgs



No lepton present
VEV only, no Higgs present
Lepton present
Higgs present and VEV

Bound states of quarks

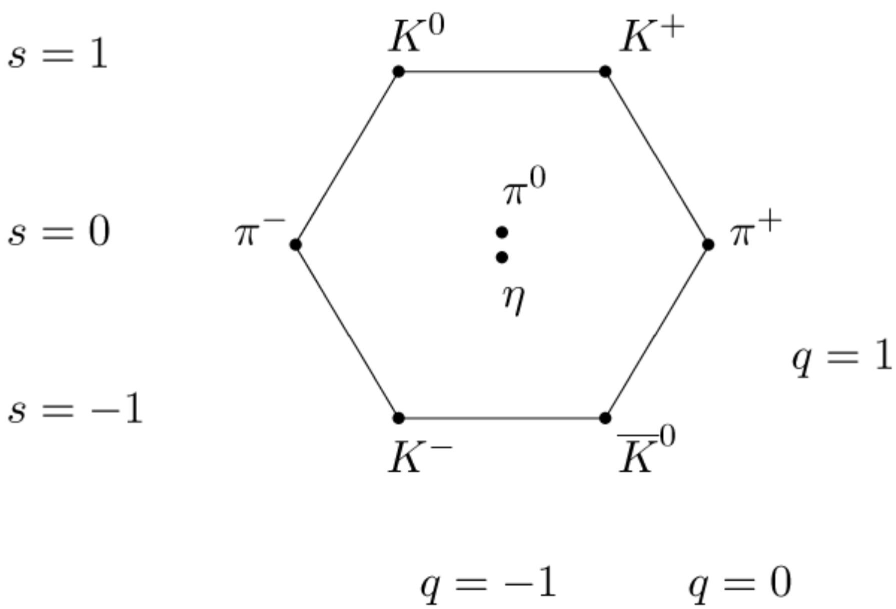
Mostly

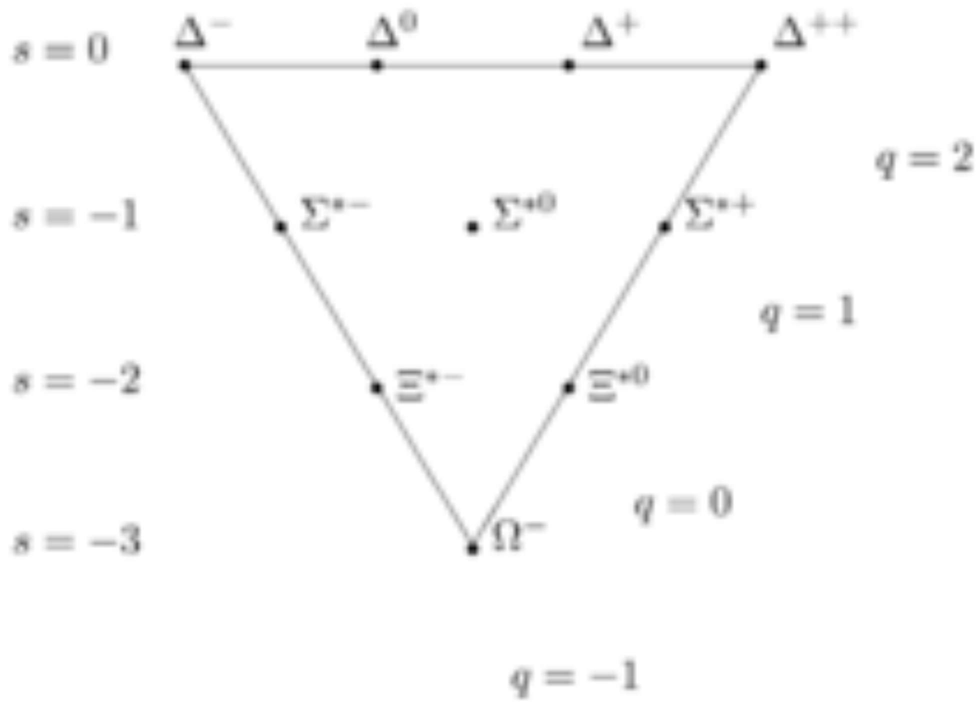
qqq Baryon – protons and neutrons

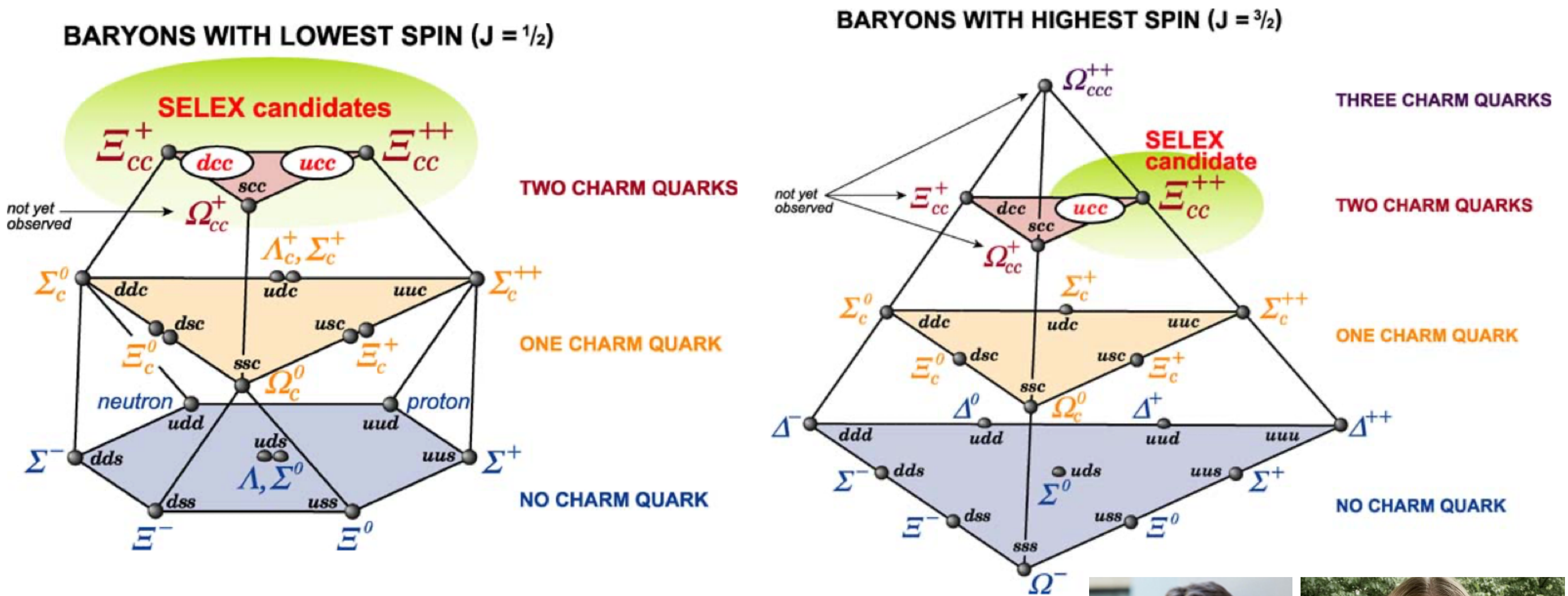
$\boldsymbol{q\bar{q}}$ Meson – pions

q can be *u, d, s, c, or b*. Top quark decays before a meson can form.

Organization of mesons and baryons

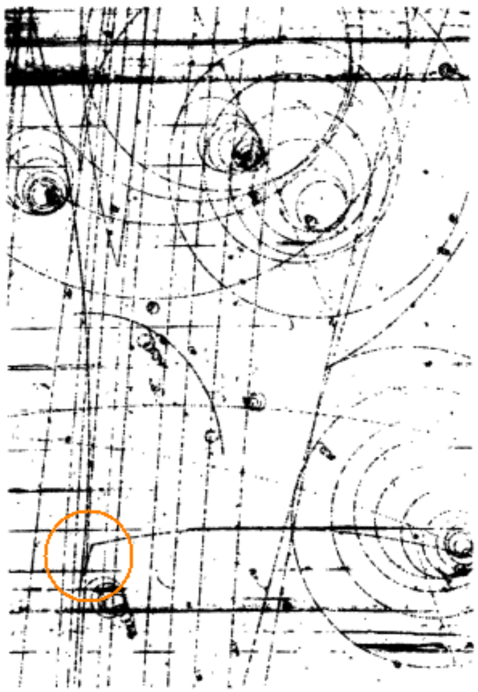


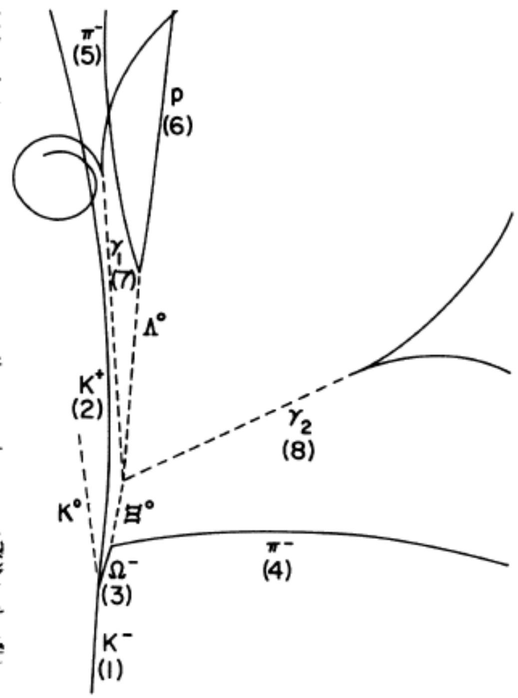




After 35 years of effort, can compute particle properties with massive CPU resources

Will Detmold, Phiala Shanahan at MIT





$$K^- + p \rightarrow \Omega^- + K^+ + K^0$$

$$\Omega^- \rightarrow \Xi^0 + \pi^-$$

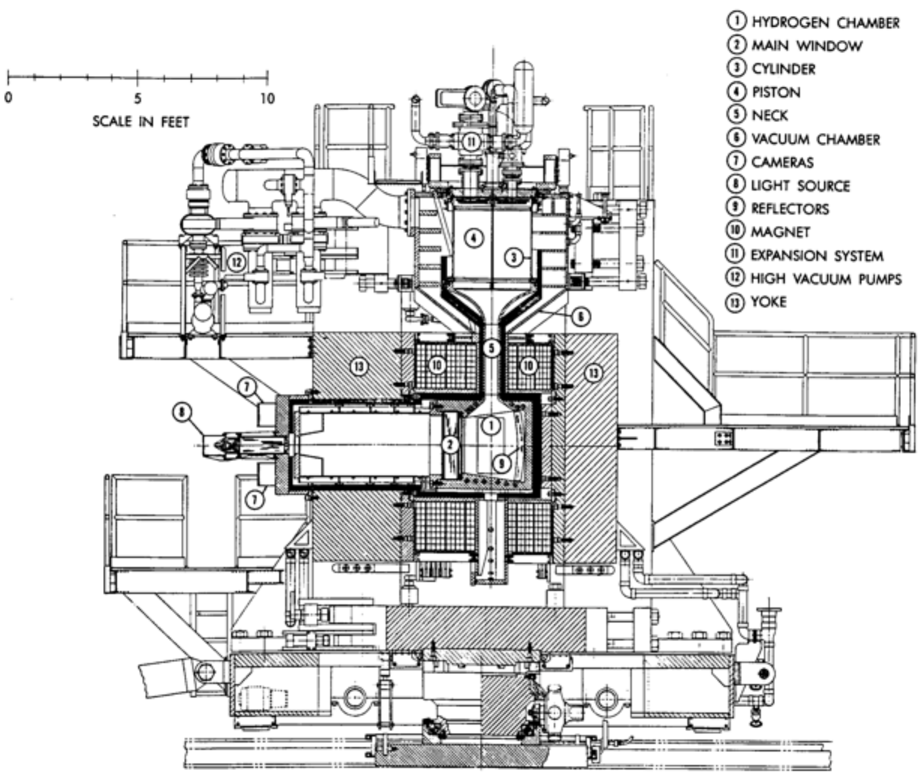
$$\Xi^0 \rightarrow \Lambda^0 + \pi^0$$

$$\pi^0 \rightarrow \gamma_1 + \gamma_2$$

$$\gamma_2 \rightarrow e^+ + e^-$$

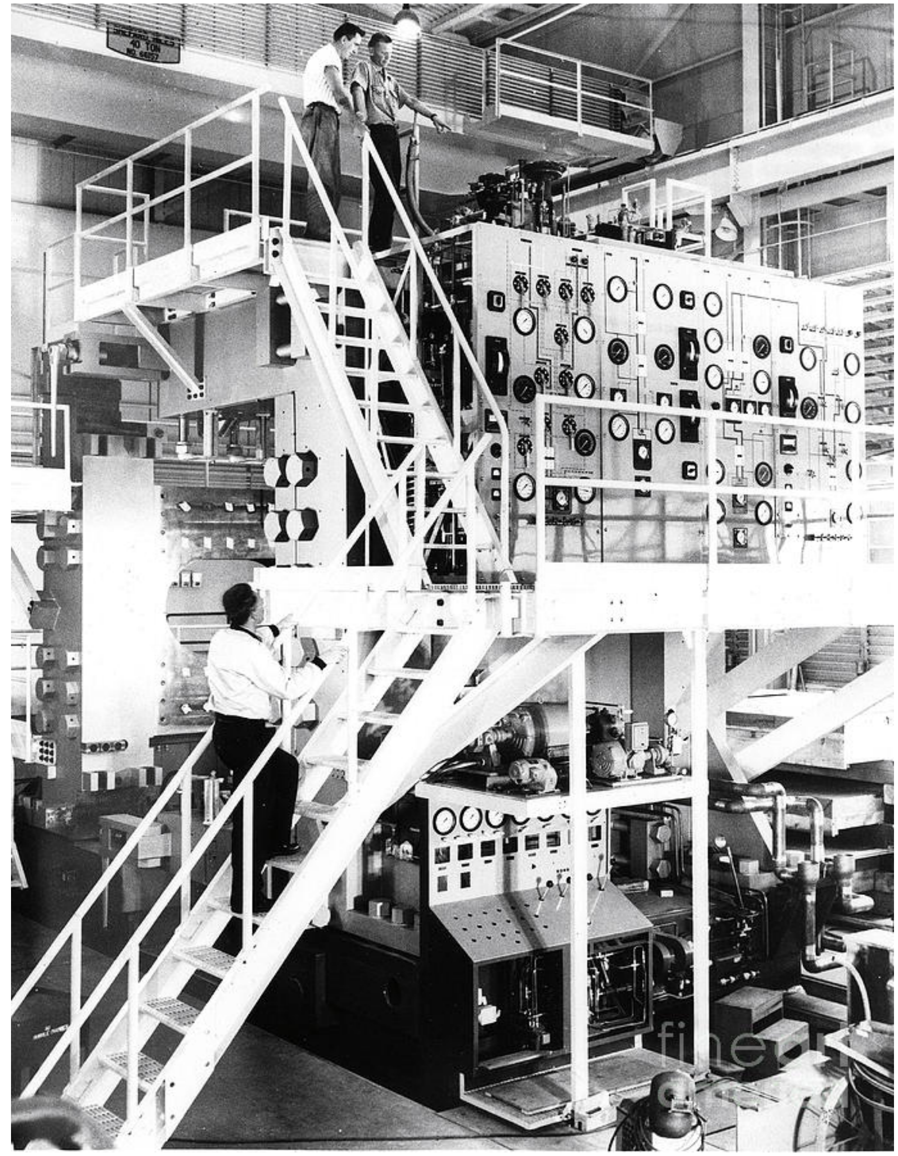
$$\gamma_1 \rightarrow e^+ + e^-$$

$$\Lambda^0 \rightarrow \pi^- + p.$$



Schematic cross section of the 80-inch liquid hydrogen bubble chamber showing major components.

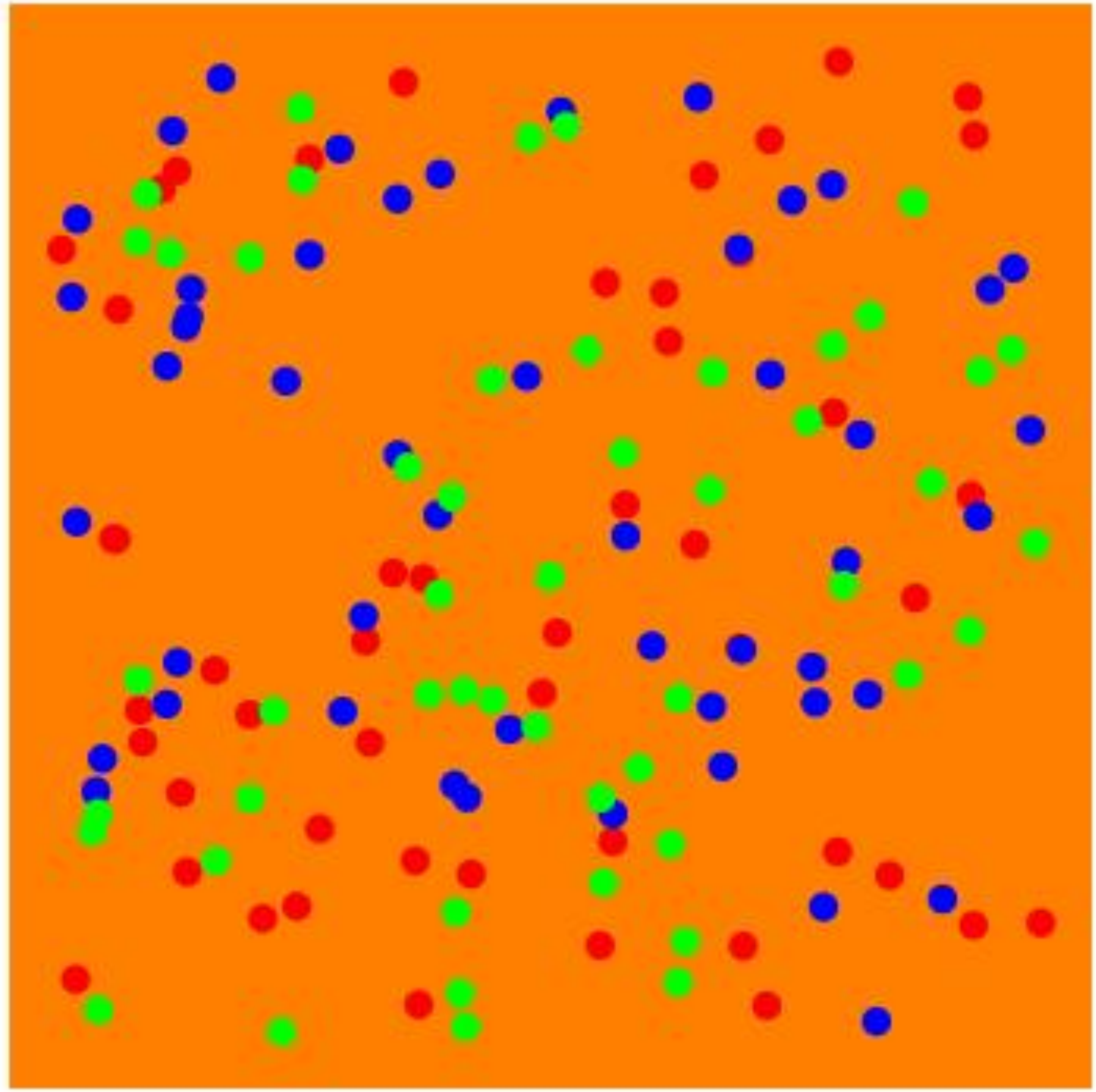
BNL 80 inch Bubble Chamber



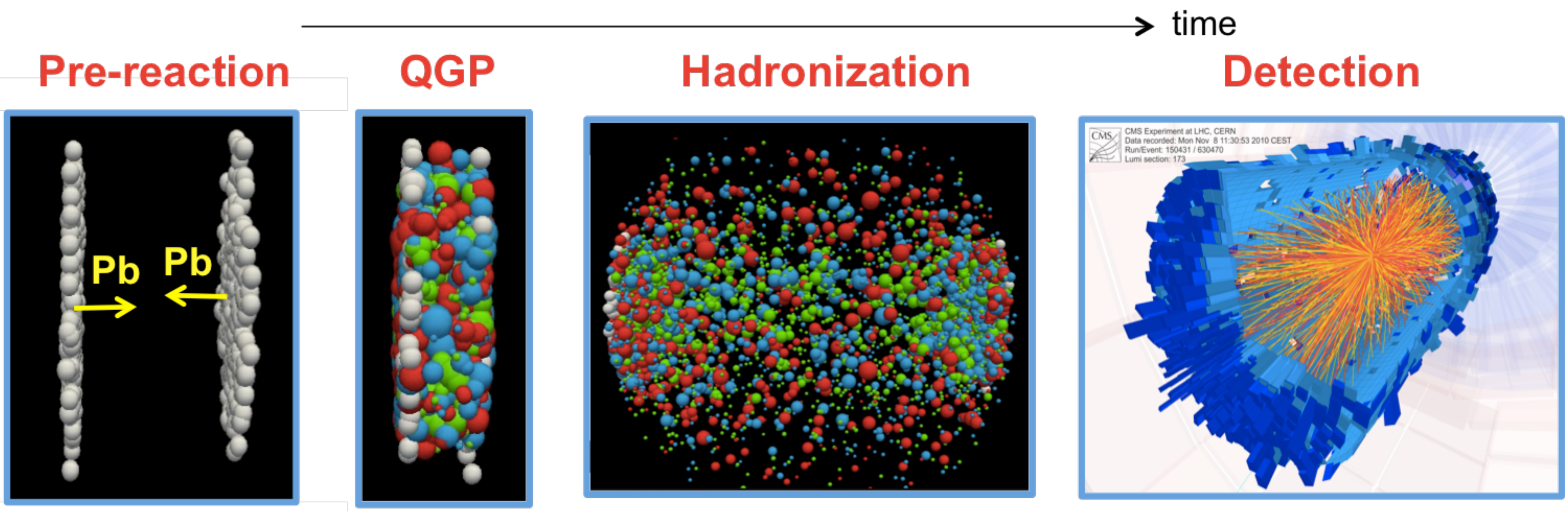
Electro-magnetic plasma

Quark Gluon Plasma

The gluons (strong equivalent of photons) interact with *each other* and create a "paste" or "soup" that the quarks have to survive in.



QGP created and studied at CERN



Barbara Jacak (Berkeley)

Yen-Jie Lee (MIT)

Early timeline in the Modern Era

Process			
EW	8.2 ps	3 Q K	W, Z Fall out of equilibrium, EM and weak separate
Nucleon-quark	0.8 μs	10G K	Quarks fall out of equilibrium, nucleons annihilate with anti-nucleons
Nucleon-pion	0.7 μs	12G K	
Pion-quark	30 μs	2G K	Pions fall out of equilibrium with quarks, quickly decay

What happens before 8 ps depends on a theory we do not yet know!