MEMORANDUM

To: Self

From: Peter Fisher

Subject: Filling in "Enhanced Sensitivity of Photodetection iva Quantum Illumination"

Date: January 26, 2019

---

1 Unentangled illumination, $b << 1$

System operates over a bandwidth of W and the detector has an integration time T, with a total of $d = WT$ modes. In this regime, T must be kept small enough so that the total number of background photons b is much less than one. A signal photon ρ is sent in mode k in d and $\rho = |k\rangle\langle k|$. The signal photon travels to the object, is reflected and may or may not be detected.

Each mode is subject to background and,

$$\rho_j = (1-b)\,|vac\rangle\langle vac| + b\,|j\rangle\langle j| \ \text{ if } j \neq k \quad (1)$$

$$= |j\rangle\langle j| \ \text{ if } j = k \quad (2)$$

If the returning signal photon is not reflected, the detector sees

$$\rho_o = \rho_1 \otimes \ldots \otimes \rho_d \quad (3)$$

$$= (1-b)^d\,|vac\rangle\langle vac| + b\sum_{k=1}^{d}|k\rangle\langle k| + b^2\sum_{k=1}^{d}\sum_{l=1}^{d}|k\rangle|l\rangle\langle l|\langle k| + \ldots \quad (4)$$

The notation $|k\rangle|i\rangle$ refers to two unentangled photons arriving at the detector in the same time interval T. Discarding terms in Eq. 4 of order b^2 and higher leaves,

$$\rho_o = (1-db)\,|vac\rangle\langle vac| + b\sum_{k=1}^{d}|k\rangle\langle k|\,.$$

References

[1] Lloyd, S.,"Enhanced Sensitivity of Photodetection via Quantum Illumination", Science, 321(2008)1463.