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MEMORANDUM

To: Scott

From: Peter

Subject: Relative risk

Date: May 29, 2020

Aerosol transmission may be an important part fo the way the SAR-CoV-2 virus spreads. The papers by Doyle, Kemp, and Evans provide a means of assessing relative risk to aerosol transmission for those working or learning in labs with fresh air changes or filtering. This note provides a method for computing the relative risk for different numbers of people in a room and an example using the method of Evans in order to assess risk as a function of parameter variations.

A research group has N people and the PI must decide how many, m, will work at the same time in a laboratory. f is the poorly known infection rate among the N researchers in the group and $t_\circ$ is the time between infection and the appearance of symptoms. Reducing the number of transmissions of the SAR-CoV-2 virus between group members while working in the lab is the PI's goal. The time rate of transmission in the lab, g may be calculated using the Evan's method and depends on air-changes in the room, the virus lifetime as an aerosol, and so on. A workday in the lab is τ.

If $m = 0$ or $m = 1$, transmission cannot take place. If the PI chooses to have two people in the room, then the probability none are infected is $(1 - ft_\circ)^2$, that one is infection $2\,(1 - ft_\circ)\,ft_\circ$, and that both are infected $(ft_\circ)^2$. Transmission occurs if one researcher is infected and the other is not and the probability is $g\tau$. The combined probability for transmission to occur if the PI chooses to have two people in the lab is $2\,(1 - ft_\circ)\,ft_\circ g\tau$ for one day of work.

In general,

$$P\,(n\,\text{chosen}|m\,\text{infected}) \quad = \quad P\,(n|m) = \binom{n}{m} ft_\circ^m\,(1 - ft_\circ)^{n-m}\,.$$

With m infected and $n - m$ uninfected, there are $m\,(n - m)$ ways transmission can occur and $g\tau\,(1 - g\tau)^{n-1}\,m\,(n - m)$ gives the probability for one transmission. If $ng\tau << 1$, then two transmissions are unlikely compared to one transmission and the probability is $g\tau m\,(n - m)$

$$P\,(\geq 1\,\text{transmission}n\,\text{chosen}|m\,\text{infected}) \quad = \quad P\,(1, n|m) = \binom{n}{m} ft_\circ^m\,(1 - ft_\circ)^{n-m}\,g\tau m\,(n - m)$$

and summing over m gives,

$$P\,(\geq 1\,\text{transmission}n\,\text{chosen}) \quad = \quad P\,(1, n) = \sum_{m=1}^{n-1} \binom{n}{m} ft_\circ^m\,(1 - ft_\circ)^{n-m}\,g\tau m\,(n - m) \qquad (1)$$

No. in room, u	$P_{rel}(1,u)$	$P(1,u)$ 550 seat lecture	400 sq. ft. lab
2	1	2.6×10^{-6}	0.00013
3	3	7.7×10^{-6}	0.00038
4	6	0.00015	0.00076
5	10	0.000025	0.00113
10	45	0.00011	0.0057
30	435	0.0011	0.055
100	4,950	0.012	> 1 transmission

Table 1: Relative and absolute infection rates per hour. For the absolute probabilities, the number gives the probability of one transmission per hour, > 1 transmission means more than one transmission per hour is likely.

Parameter	Lab Room 400 sq. ft.	Lecture Room 551 seats, 5,728 sq. ft.
r_{src}	1 nL/min	1 nL/min
r_{room}	445 m^3/h	21,000 m^3/h
t	1 h	1 h
g	0.078/h	0.0025/h
$P(1,2)$	0.00013	2.6×10^{-6}

Table 2: Model parameters for a typical lab room and large lecture room for Evans Eq. 8.

Though important parameters, f, $t_\circ$, and g remain uncertain, Eq. 1 still gives an important result for the probability of transmission relative to the probability of transmission with just two people in the room,

$$P_{rel}(1,u) = \frac{P(1,u)}{P(1,2)} = \frac{1}{2}u(u-1) \tag{2}$$

Table 1 gives the increased relative risk as people are added to the room.

The absolute probability $P(1,2)$ depends on the infection rate between two people g, the infection rate of the research group f, and $t_\circ$, the time between infection and symptoms. In the state of Massachusetts, there are currently 1,000 new cases per day for a population of 6.7 million. The daily new cases result from testing, giving a lower limit of $f > 0.00015$/day as many infections go untested and unreported. $t_\circ = 5.1 \pm 0.7$ days (https://www.acpjournals.org/doi/10.7326/M20-0504). Evans Eqs. 1 and 8 gives g for a specific room. Table 2 gives the model parameters for a typical lab and 500 seat lecture room at MIT.

Table 1 gives the absolute probabilities per hour of exposure for a lab and lecture hall in he right columns. If researchers in a lab so not have much contact outside of the group for $\approx$ 2 weeks or 80 hours, confidence builds that they are not and unlikely to become infected. For five people working together for 80 hours have a group probability of 9% of becoming infected. For a large lecture hall, a term of lectures is 42 hours of lecture (14 weeks, 3 hours per week), so a group of 100 people have a probability of 40% of transmitting one infection between two members.

$P(1,2)$ has a linear dependence on f, $t_\circ$, and $_{src}$ because of the small transmission rate. Eqs. 3-4 gives expansions for labs and lecture halls around the nominal values. In combination with Eq. 2, Eq. 2 gives the single transmission probability for parameter value within a factor of ten of the nominal values.

$$P(1,2)_{lab} = 0.00013\left[1 + (r_{src} - 1\,\mathrm{nL/min}) + (f - 0.00015/\mathrm{day}) + (t_{\circ} - 5.1\mathrm{day})\right] \quad (3)$$

$$P(1,2)_{26-100} = 2.6 \times 10^{-6}\left[1 + (r_{src} - 1\,\mathrm{nL/min}) + (f - 0.00015/\mathrm{day}) + (t_{\circ} - 5.1\mathrm{day})\right) \quad (4)$$